

A Search for the Decay $\mu^+ \rightarrow e^+ \gamma$

INAUGURAL-DISSERTATION

zur Erlangung der Philosophischen Doktorwürde
vorgelegt der Philosophischen Fakultät II
der Universität Zürich

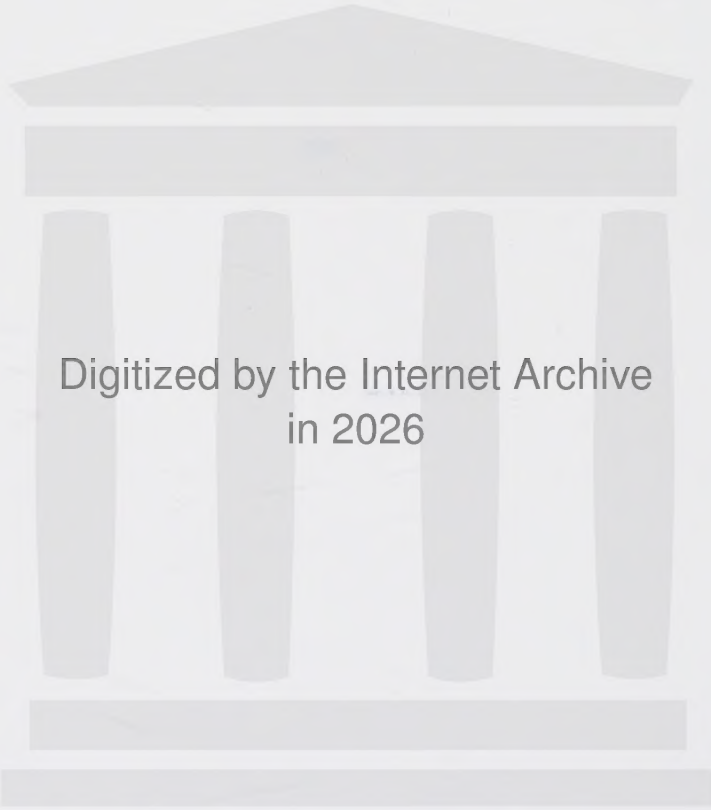
von

ANDRIES VAN DER SCHAAF
aus Amsterdam

Begutachtet von Herrn Prof. Dr. R. Engfer



Juris Druck + Verlag Zürich
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1. INTRODUCTION

Neutrinoless $\mu \rightarrow e$ transitions (e.g. $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$, $\mu^- A \rightarrow e^\pm A^*$ and $\mu \rightarrow e\gamma\gamma$) have not been seen up to now. This work describes a search for the decay $\mu^+ \rightarrow e^+ \gamma$ performed at SIN. No evidence for the existence of the process has been found. An upper limit for the branching ratio relative to the normal decay $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$ of 1.0×10^{-9} (90% confidence) will be presented.

As an introduction to the subject, a historical review, a formal definition of the concept of muon number conservation that forbids these processes and a discussion of some ideas about the nature of that principle and its possible violation will be given. The description of the experiment includes discussions of the detection system, the electronics, the data reduction and interpretation and the final result. Some technical aspects and a table with the experimental limits on some muon number violating processes are presented in the appendices.

1.1 History

The interest in the decay $\mu \rightarrow e \gamma$ and other neutrinoless $\mu \rightarrow e$ transformations is directly related to the question: why is there a muon at all? The only classical distinction between electron and muon is the difference in mass. In weak interactions with hadrons the muon acts with the same strength as the electron. This "μ-e universality" is tested at a 1% level by the branching ratio $\Gamma_{\pi \rightarrow \mu \nu} / \Gamma_{\pi \rightarrow e \nu}$ (1). The fact that the muon, considered as a heavy electron, cannot be described by an electromagnetic excitation of the electron was discussed by G. Feinberg et al. in 1959 (2). An electromagnetic de-excitation of the muon would have completely dominated the weak decay mode. A few years later, in 1962, it was shown experimentally by G. Danby et al (3) that the neutrino associated with the muon does not interact with the electron. This result is a direct proof that there exist two fundamentally different neutrinos, ν_e and ν_μ , and is a strong support for viewing the muon as a fundamental particle and for the introduction of a new quantum number: the muon number. A hint in this direction had been given already by a calculation by Feinberg in 1958 (4) of the rate for $\mu^+ \rightarrow e^+ \gamma$ under the assumption of an intermediate boson as the propagator of the weak interaction. The result of this calculation for the branching ratio of $\mu^+ \rightarrow e^+ \gamma$ ($\sim 10^{-4}$ in the case $\nu_e = \nu_\mu$) disagreed with the existing upper limit of 2×10^{-5} (90% confidence) measured in 1955 by S. Lokanathan et al (5).

The experimental upper limit decreased rapidly to 2.2×10^{-8} (Parker et al, 1963, (6)), after which the interest in the process weakened for about a decade.

The construction of the so-called meson factories in the beginning of the 1970's triggered a new generation of $\mu^+ e^+ \gamma$ searches (7,8,9,10), one of which is the subject of this work.

The revival of experimental activities coincides with an increased theoretical interest in the concept of muon number conservation. The development of unified theories of weak and electromagnetic interactions in which muon number conservation in general is not a fundamental law (like, for example, charge conservation) led to a series of estimates for the rates of the different muon number non-conserving processes. In the original Weinberg-Salam model of 1967 (11) muon number conservation holds exactly, but recent experimental information, such as the discovery of the τ lepton, indicates that this minimal model has to be extended. In most generalizations muon number conservation is no longer a natural phenomenon, although none of the schemes is able to predict the rate of one of the "forbidden" reactions unambiguously.

More sensitive to the properties of the different models are the branching ratios between the various processes and their knowledge would help to discriminate between the schemes.

1.2 Muon number conservation phenomenology

There are several lepton schemes that describe the non-occurrence of the various $\mu \rightarrow e$ transformations that are allowed kinematically. It should be emphasized that these schemes neither explain these phenomena nor do they give a deeper understanding. They merely provide us with an economical method for organizing the experimental information. A more fundamental treatment can be given within the framework of explicit models, some of which incorporate quite naturally mechanisms that violate muon number conservation. They will be discussed in the next section.

Table 1 presents the three lepton schemes that are commonly referred to in the literature. By far the most popular is the additive scheme. The muon and its neutrino have the same lepton numbers as the electron and its neutrino, but there is an additional muon number that is zero for the electron and its neutrino, +1 for the muon and its neutrino, and -1 for their anti-particles.

The Konopinski-Mahmoud scheme (12) is the most economical in the sense that it does not introduce a new quantum number, but distinguishes the particles by charge and helicity. The neutrinos are postulated to be massless in this scheme.

The multiplicative scheme postulates as additional parameter a lepton parity (13), that is +1 for the electron and its neutrino and -1 for the muon and its neutrino. The anti-particles have

Additive scheme:

L_μ L		0		+1	-1	
+1		$e^- \nu_e$		$\mu^- \nu_\mu$		$\sum L$ constant
-1		$e^+ \bar{\nu}_e$			$\mu^+ \bar{\nu}_\mu$	$\sum L_\mu$ constant

Konopinski Mahmoud:

H L		+1		-1	
+1		$e^- \mu^+$	$\bar{\nu}_\mu$	ν_e	$\sum L$ constant
-1		$e^+ \mu^-$	$\bar{\nu}_e$	ν_μ	

Multiplicative scheme:

L_p L		+1		-1	
+1			$e^- \nu_e$	$\mu^- \nu_\mu$	$\sum L$ constant
-1			$e^+ \bar{\nu}_e$	$\mu^+ \bar{\nu}_\mu$	$\prod L_p$ constant

Table I: Lepton number schemes

the same parity as the particles.

All schemes completely forbid all $\mu \rightarrow e$ transformations, with one exception. Charge changing processes, like $\mu^-(A, Z) \rightarrow e^+(A, Z-2)$, are allowed in the Konopinski-Mahmoud scheme, but they turn out to be strongly suppressed dynamically because of the V-A structure of the weak interaction.

1.3 Muon number conservation in the framework of unified gauge theories

At present one of the major activities of theoretical elementary particle physics is the attempt to describe the weak, the electromagnetic and the strong interaction as different manifestations of one fundamental " unified " interaction. The different manifestations are the consequence of the differences in properties of the particles that propagate the interaction. The $1/r^2$ dependence of the electromagnetic interaction is caused by the zero mass of its propagator, the photon. The pointlike and much weaker weak interaction is explained as being mediated by massive propagators ($W^\pm, Z^0$). Because the only available succesful interaction theory is quantum electrodynamics, all efforts go towards a generalization of this theory to a non-Abelian gauge theory. A detailed treatment of these types of theories is given in ref.14,15,16.

In the theory of electrodynamics the Lagrangian is invariant under a local gauge transformation of the fields ($\phi_1^*(x) = \exp\{-iq_1\theta(x)\}\phi_1(x)$), because the photon is massless. In order to describe interactions that are mediated by massive particles one assumes that although the Lagrangian is gauge invariant, mass terms are introduced by a so-called spontaneous breaking of the symmetry. In the formal treatment of the theory the

gauge fields get their masses by "absorbing" the Goldstone bosons that originate in the symmetry breaking (Higgs mechanism). Some of these bosons might be real and play a role in some classes of muon number violating theories.

Foremost among the benefits of this theory, compared to a conventional current-current type of weak interaction theory, is its renormalizability. Higher order diagrams can be calculated without the use of arbitrary cut-off parameters.

The theory has been extended to the strong interaction (17) which is described by gluon exchange between quarks. The principle of quark-lepton analogy led to the prediction of the fourth (charmed) quark that was found in 1974 (18). Other successful predictions are the neutral weak interaction, (verified in 1973 (19)) and parity violation in atoms (found very recently in 1979 (20)).

Most models predicting muon number violation postulate the existence of extra particles that would be very hard to detect directly. One exception is the ν_e - ν_μ mixing model, first proposed by B. Pontecorvo in 1958 (21). Predicted branching ratios for the various $\mu \rightarrow e$ transitions range from zero to 10^{-26} (22), depending on the neutrino mass difference and the mixing angle. These numbers are far too small to be measurable with present day technology. More sensitive to the properties

of this model are searches for neutrino oscillations.

The other models can be classified according to the type of extra particle that has been postulated to produce the effect:

- A: extra Higgs bosons (23),
- B: left-handed heavy neutral lepton(s) (24,25,26,27),
- C: doubly-charged leptons (28),
- D: right-handed heavy neutral leptons (29,30,31,32,33)

All these models are able to explain a branching ratio for $\mu^+ \rightarrow e^+ \gamma$ that lies between zero and the present experimental upper limit by properly adjusting the mass of the new particle and some mixing parameter. Detailed comparisons between the different schemes can be found in references (34,35,36).

The models have more predictive power with respect to the branching ratios between the different muon number violating processes, since these depend only weakly on the masses and are independent of the mixing parameters. In general the rate for nuclear muon conversion tends to be higher than that for $\mu^+ \rightarrow e^+ \gamma$ whereas (except for case C) the rate for $\mu^+ \rightarrow 3e$ is expected to be lower.

2. EXPERIMENTAL PROCEDURE

2.1. General considerations

The decay $\mu^+ \rightarrow e^+ \gamma$ is characterized by the emission of a positron and a photon moving back to back with energies $E_e = E_\gamma = \frac{m_\mu}{2} = 53 \text{ MeV}$. Background arises mainly from the radiative decay $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \gamma$ and from accidental coincidences between positrons from the normal muon decay $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$ and bremsstrahlung photons. True coincident positrons and photons from the radiative decay can be suppressed only by sufficient energy and angular resolution. This decay serves as a normalization and has in addition its own interest as a test of the V-A structure of the weak interaction. An analysis of the data in terms of the weak interaction coupling constants as it was previously done (37) is under way (38). In the region of interest accidental coincidences are the most severe background source and can be reduced by good energy and angular resolution as well as good timing between the photon and positron.

In chapter 2 the experimental procedure is described including beam, detectors, electronics and data acquisition system. Chapter 3 is devoted to the data reduction to 2-dimensional positron-photon energy spectra. Chapter 4 deals with the analysis of these spectra and finally in chapter 5 the result is presented.

The aim of the experiment was to search for the decay $\mu^+ \rightarrow e^+ \gamma$ with at least an order of magnitude higher sensitivity than previously achieved. In designing a detection system for this purpose one encounters the problem of optimizing simultaneously the efficiency and the signal-to-background ratio of the experiment. Given a stop rate of $5 \times 10^5 \mu^+/\text{s}$ one needs about 1% acceptance to reach an upper limit of 10^{-9} (90% confidence) during a one-week experiment. This consideration unavoidably leads to the choice of a NaI(Tl) shower detector as the photon detector. The alternative of a pair spectrometer with its superior energy and angular resolution suffers from an insufficient detection probability. The reason for using also a NaI(Tl) crystal as the main positron detector is, besides its simplicity, the limited importance to achieve the ultimate positron energy resolution offered by a magnetic spectrometer. One argument is the 4 MeV uncertainty in the target energy loss, a second that the accidental background can be reduced only linearly with the positron energy resolution because of the constant intensity at the upper end of the $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$ spectrum (Michel spectrum).

The detection system is shown in fig. 1 and will be described in detail in section 2.2. The electronics and the data-acquisition system will be discussed in section 2.3.

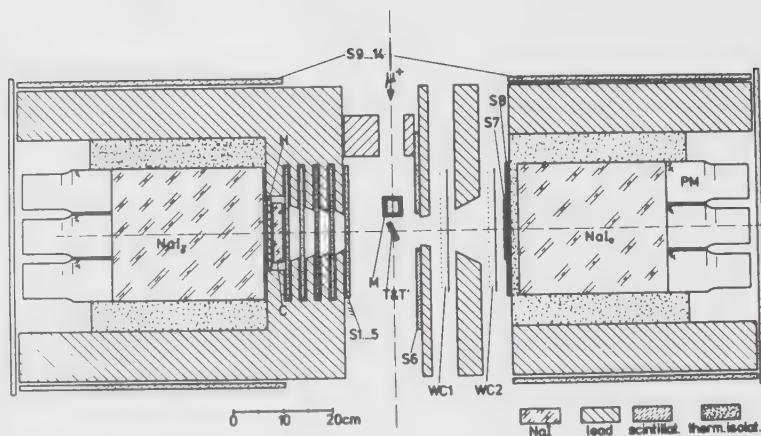
The actual measurements and the associated calibration runs were performed in 1977 during four periods of ten days at the $\mu E1$ superconducting muon channel at SIN. The beam was tuned to 90 MeV/c and had π^+ and e^+ contaminations below 1% and 1% respectively. In order to keep the accidental background and the electronics dead time at an acceptable level the beam intensity was limited to $2-3 \times 10^6 \mu^+/s$ giving a stop rate of $4-6 \times 10^5 \mu^+/s$. The total number of stopped muons during the actual data taking was 6×10^{11} . To meet the requirements of another experiment during half of the runs, the accelerator was operated with a 400 kHz, 30% duty cycle beam structure leading to a fluctuating muon decay time distribution. For the rest of the time the beam was continuous with 100% duty cycle recommendable to minimize accidental coincidences. During part of the experiment the channel was tuned to stop negative pions in a LiH target. The decay $\pi^0 \rightarrow 2\gamma$ of the produced π^0 's was measured for calibration purposes. In between the runs, without changing the geometry of the detection system, we measured cosmic muons traversing both detectors and the target to calibrate the collinearity test.

To complete the energy calibrations from $\pi^0 \rightarrow 2\gamma$ and $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \gamma$ as discussed in ref. (8) we studied the dependence of the response functions of the NaI(Tl) crystals on the impact position and angle of the detected particle in the energy range 20 - 150 MeV with a collimated monosenergetic positron beam (see section 4.2).

2.2. The detection system

The experimental arrangement is shown in Fig. 1. The main components are two cylindrical NaI(Tl) crystals NaI_e and NaI_γ (277 mm $\varnothing$, depth 220 mm) for the energy measurements. The wire chambers WC1 and WC2 in front of NaI_e together with the converter-hodoscope system in front of NaI_γ enable a collinearity test on those data for which the photon converts. The collinearity test is applicable for only a quarter of the events at 50 MeV photon energy owing to the limited detection probability of the converter C. It was mainly intended to be an additional check in the case of a positive $\mu^+ \rightarrow e^+ \gamma$ signal rather than to reduce the background.

The muon beam is degraded in counter M before it reaches the target counters T and T'. Counter M is used to veto events that are promptly beam correlated. Its central part is made of material capable of re-emitting the absorbed Cerenkov radiation isotropically with a spectrum matched to the spectral sensitivity of the photocathode. This part, sensitive especially to the positrons in the beam, is covered by 2 mm scintillator material in order to raise the sensitivity to non-relativistic beam particles and positrons from muons that stopped at its surface. To reduce the amount of passive material in the stop region, T and



1. Experimental set-up

M	: Cerenkov counter	(40 × 40 × 40 mm ³)
T, T'	: target counters	(40 mm Ø, 5 mm each)
S1-S6	: veto counters	(10 mm each)
S7	: positron trigger counter	(120 mm, 5 mm)
S8	: pile-up veto counter	(277 mm Ø, 5 mm)
S9-S14	: cosmic ray veto counters	(10 mm)
	S15-S18	not shown
C	: NaI(Tl) converter disk	(120 mm Ø, 20 mm)
H	: hodoscope	(two planes, 2 × 10 strips, 14 mm width, 3 mm)
WC1 & WC2	: multi-wire proportional chambers	
NaI _δ & NaI _γ	: NaI(Tl) crystals	(277 mm Ø, 330 mm each)

T' are coupled to their photomultipliers by air light guides. The use of two targets allows for a partial correction of the positron energy loss. We used targets as small as possible in order to maximize the acceptance for collinear events. This acceptance varies from 1.9% in the center to 1.2% at the edge of the targets. As only 20% of the muons reaching M stop in T or T' a considerable number of them come to rest in material around the targets, mainly in M, S5 and S6. These counters are sensitive to the positrons from muon decays and can therefore also reduce the photon background since most photons are accompanied by positrons.

The positron detector is composed of a cylindrical crystal NaI_e viewed by seven photomultipliers (75 mm $\varnothing$, rise-time 2 ns), two plastic scintillation counters S7 and S8 and two multiwire proportional chambers WC1 and WC2. S7 defines the solid angle of the detector (1.9%) for the on-line trigger and gives the positron timing signal. S8 is used to reject pile-up of two positron signals within the 400 ns integration time of the pulses of NaI_e . Both wire chambers consist of two planes of 64 anode wires with 2 mm spacings and cathode planes made of aluminium foil. The information of the wire chambers allows for a collinearity test for those events of which the photon direction is known. The selection of single tracks pointing to the targets rejects scattered positrons. It is furthermore possible to correct for the position dependence of the S7 timing, to study the response function of NaI_e for different impact positions and to correct for the target

energy loss as a function of the effective target path. Two lead collimators (total thickness 70 mm) reduce neutral pile-up and keep the count-rates in WC1, WC2 and NaI_g at acceptable levels. The outer collimator is covered by counter S6 with a hole of 80 mm $\varnothing$.

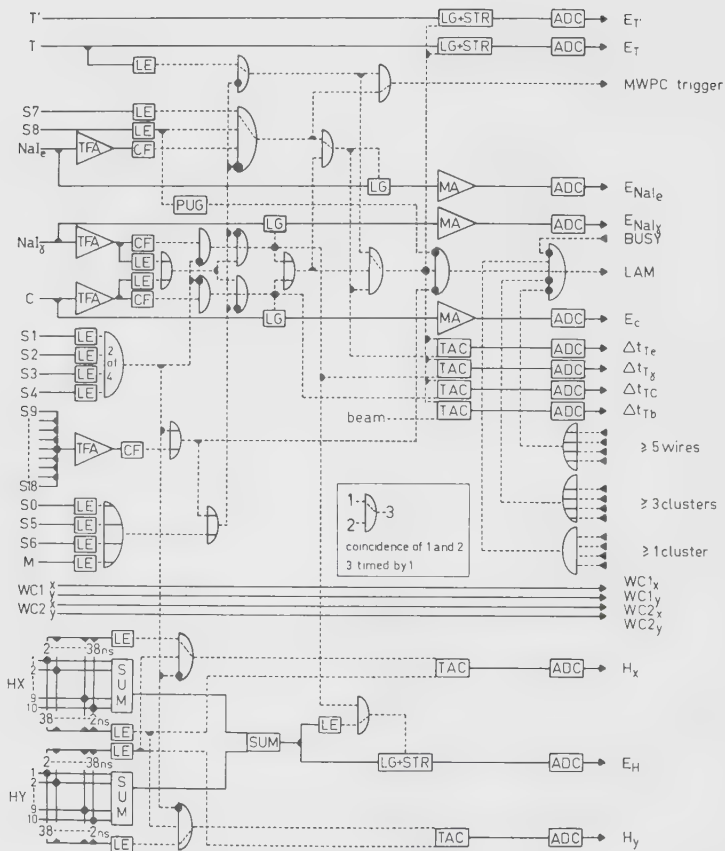
The photon detector consists of the crystal NaI_γ viewed by seven photomultipliers (75 mm $\varnothing$, rise-time 10 ns), a converter-hodoscope system and a set of five veto counters. The rise-time of the photomultipliers of NaI_g and NaI_γ is of minor importance since it is small compared to the rise-time of about 60 ns caused by the geometry of the crystals and their windows. The converter C is a NaI(Tl) disk of 20 mm thickness coupled to one photo-multiplier. Its photon conversion probability at 53 MeV amounts to 27%. 90% of the converted photons give signals in both planes of the hodoscope H. The crystals NaI_γ and C have 0.2 mm thick aluminium windows resulting in negligible energy loss. H consists of two planes of ten plastic scintillator strips of 14 mm width each. We preferred a thin hodoscope to a practically massless but thicker wire chamber because both the energy resolution and the position resolution strongly depend on the distance between NaI_γ and C. The counters S1 - S5 are sandwiched between lead collimator rings. They veto charged particles and restrict positrons misidentified as photons to the energy region below 43 MeV due to their energy loss in S1 - S5.

The lead collimators are designed in such a way that no surfaces are seen by the target and the detectors simultaneously. Both crystals are surrounded by 10 cm lead, 2 mm cadmium and about 50 cm borated paraffine. The counters S9 - S18 and 1.5 m heavy concrete above the apparatus reduce the cosmic ray background by a factor of fifteen in the region of 50 MeV deposited energy. The three NaI(Tl) crystals, the targets, S7 and S8 are equipped with light emitting diodes (LED's) to monitor the stability and dead time of the electronics.

2.3. Electronics and data acquisition

A block diagram of the electronics is shown in Fig. 2. The signals on the left side originate from the various detectors whereas the right side shows the information recorded on magnetic tape. Measured are the energies E_{NaI_e} , E_{NaI_γ} , E_C , E_H , E_T and $E_{T'}$, the coordinates of the hodoscope H_X and H_Y and of the wire chambers $WC1_{X,Y}$ and $WC2_{X,Y}$ and the time differences Δt_{eT} , $\Delta t_{\gamma T}$, Δt_{CT} and Δt_{bT} between T and S7, NaI_γ , C and a beam counter respectively.

The master trigger LAM for data read out through a CAMAC system into the computer is defined as the coincidence within 100 ns of the time signals from the target T, the positron detector



2. Block diagram of the electronics

Solid lines represent analog data lines, dashed lines represent logic data lines. The labeling of the input signals on the left corresponds to that of the detectors in fig. 1. The digitized information on the right side is recorded on magnetic tape. The labeling corresponds to that of fig. 3.

ADC: analog to digital converter, CF: constant fraction timing discriminator, LAM: trigger for data read-out (look at me), LE: leading edge timing discriminator, LG: linear gate, MA: main amplifier, MWPC: multiwire proportional chamber, PUG: pile-up gate, STR: stretcher, SUM: linear adder, TAC: time to amplitude converter, TFA: timing filter amplifier.

and the photon detector, inhibited by several veto counters and pile-up signals and the BUSY signal set during read-out. The time signal of the positron detector is the coincidence of S7, S8, NaI_e , WC1 and WC2, timed by S7. The signals of WC1 and WC2 are generated for events with one or two wire clusters and less than five responding wires per plane. The time signal of the photon detector is given by NaI_γ or C with an energy threshold of about 5 MeV.

The three detector signals are separately vetoed by each of the counters M, S1 ... S4, S5, S9 ... S18. A veto from S1 ... S4 is accepted only if at least two of them are in coincidence to avoid triggering on spurious signals and on low energy backward radiation from the crystal. Besides the rejection of promptly coincident background there is a suppression of pile-up with charged particles going through S1 ... S4, S8 or S9 ... S18 during the 400 ns integration time of the NaI(Tl) detectors.

The energy signals of the NaI(Tl) detectors are obtained by integrating the first 400 ns of the photomultiplier pulses selected by linear gates. To prevent pile-up in the main amplifiers the gates were not opened within 5 μs after the preceding signal. The energy signals of the plastic counters were obtained from the amplitudes of the photomultiplier pulses by linear gates and stretchers. The hodoscope signals are converted by delay lines (2 ... 36 ns) into two time differences measured with TAC's. Not shown in Fig. 2 are scalers and the LED electronics.

The LED signals are shaped to resemble the scintillation signals to be able to check the linear response of the analog electronics. During the measurements the LED's were pulsed with a frequency of 1 Hz, equal approximately to the event rate. Their energy and time information are recorded as a monitor for the stability and dead time of the experiment.

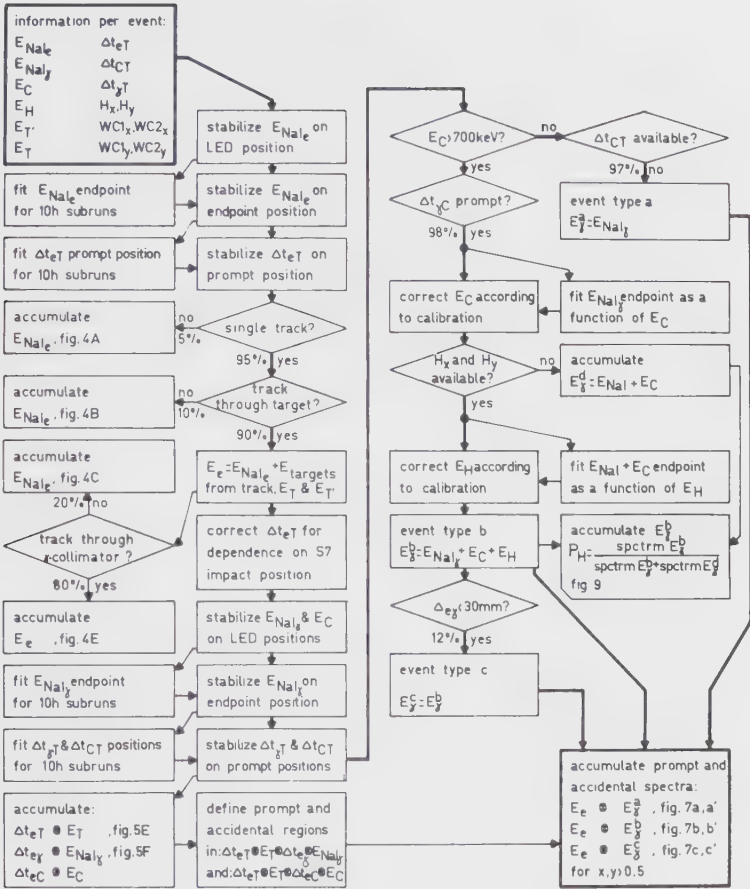
We continuously examined the behaviour of the system by accumulating several time and energy spectra, the hodoscope and wire chamber profiles and count rates. Finally the photo-multiplier pulses from the NaI(Tl) crystals were photographed and a listing of the digital information was printed for events with positron and photon energies close to 53 MeV.

3. DATA PROCESSING

A flow chart of the data processing is shown in fig. 3. The box in the upper left corner contains the information registered for each event, corresponding to the right edge of fig. 2. The lower right box contains the reduced information in the form of three sets of prompt and accidental distributions of the positron and photon energies. These sets have the following signatures:

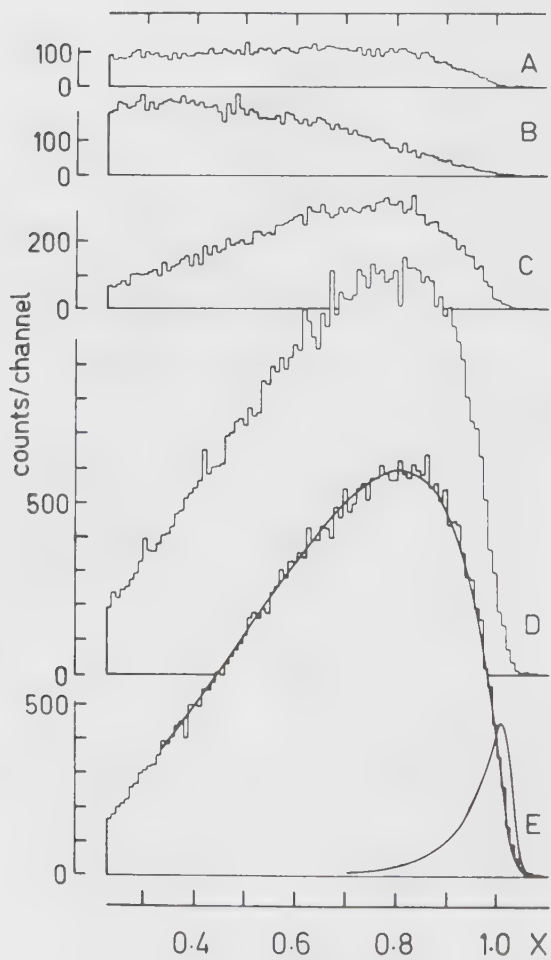
- a) The converter has no energy signal above 700 keV and no time signal: the photon did not convert.
- b) The converter has an energy signal above 700 keV, its time signal is prompt to that of NaI_Y and there is a hodoscope signal: the photon converted.
- c) The events of b) with an angle between positron and photon directions that deviates less than 7° from 180° : the photon and positron are collinear.

90% of all events are contained in one of these definitions. In the first step of the data processing the three NaI(Tl) energy signals are stabilized on the LED positions, defined as the mean positions of the preceding ten LED events. Because of the LED frequency of 1 Hz this procedure corrects instabilities on a time scale of more than a few seconds. These shifts can be related, for example, to fluctuations in the beam intensity since they are mainly caused by the rate dependence of the photo-



3. Flow chart of the data processing

The labeling of the "information per event" corresponds to that of fig. 2. The energy loss in S7, S8 and the aluminium window of NaI_e has been taken into account by offsetting the theoretical spectra.



4. Accidental energy spectrum of positrons under different positron track conditions

- A: two clusters in at least one of the four planes of WC1 and WC2
- B: one track, missing the target
- C: one track through the target, missing the entrance of the photon collimator
- D: one track through the target and the photon collimator
- E: as D, corrected for the target energy loss. A fit of the Michel spectrum with the response function indicated for $x = 1$ (FWHM = 4.3 MeV) is shown.

multiplier gains and have a range of $\pm 15\%$. To adjust for long-period drifts of the light output of the LED's, due, for example, to their temperature dependence, gain corrections of $\pm 1\%$ for time periods of about ten hours duration are determined from the endpoints of the energy spectra. The time spectra are stabilized in a similar way (range of corrections ± 1 ns).

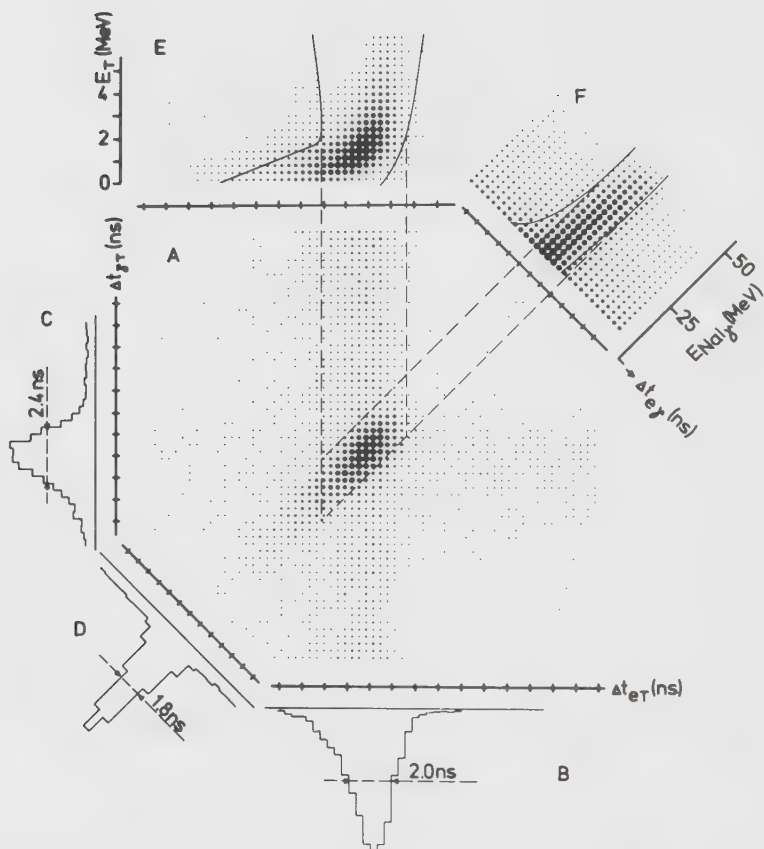
In the next step single wire chamber tracks through both the targets and S7 are selected. Fig. 4 shows the Michel spectrum under different track conditions. The distributions A and B consist of events that are rejected in the analysis. A contains double track events and B contains events with tracks missing the target or S7. The spectra C, D and E of the remaining data will be discussed in the next chapter. Since the mean energy of A is lower than that of the accepted data we conclude that there is no major pile-up contribution in A, and so verify the pile-up rejection by S8. The tracks of the events in B mostly point towards the entrance of the collimator. Both A and B consist mainly of events for which the positron from the target interacted with the collimator before reaching S7.

The positron energy E_g is the sum of the energy measured in NaI_g and the energy loss in the target. We used the measured target energies E_T and $E_{T'}$, only to decide in which of the targets the decay took place. With this knowledge and that of the measured positron direction we calculate for each individual

event the mean target path and the resulting energy loss between 0.5 and 3.0 MeV. We did not use the values of E_T and E_{γ} , themselves because they are contaminated by pile-up with a second decay positron in the case of accidental coincidences.

The photon energy E_{γ} is the sum of the energies measured in NaI_{γ} , C and H if available. E_C and E_H are calibrated such that the position of the E_{γ} endpoint does not depend on E_C and E_H . The calibrations are consistent with those from the $\pi^0 \rightarrow 2\gamma$ measurement.

The procedure of the time analysis is shown in fig. 5. Part A shows the central part of the distribution of the three time differences between the signals from the target, the positron detector and the photon detector. The positron time signal is corrected for its dependence on the impact position on S7, caused by the non-uniform light collection in S7 and the amplitude dependence of the leading edge time discriminator (range of the correction: ± 0.4 ns). Shown in parts B, C and D are the projections on Δt_{eT} , $\Delta t_{\gamma T}$ and $\Delta t_{e\gamma}$, respectively. Prompt windows are defined for Δt_{eT} and $\Delta t_{e\gamma}$ as a function of E_T and $E_{\text{NaI}_{\gamma}}$, respectively. They are shown in fig. 5E and F and contain, independently of E_T and $E_{\text{NaI}_{\gamma}}$, 98% of the true positron-target coincidences and 90% of the true positron-photon coincidences. The accidental time regions contain eight times more accidental coincidences than the prompt region. About 95% of them are prompt in Δt_{eT} and accidental in $\Delta t_{\gamma T}$. This number demonstrates the high photon

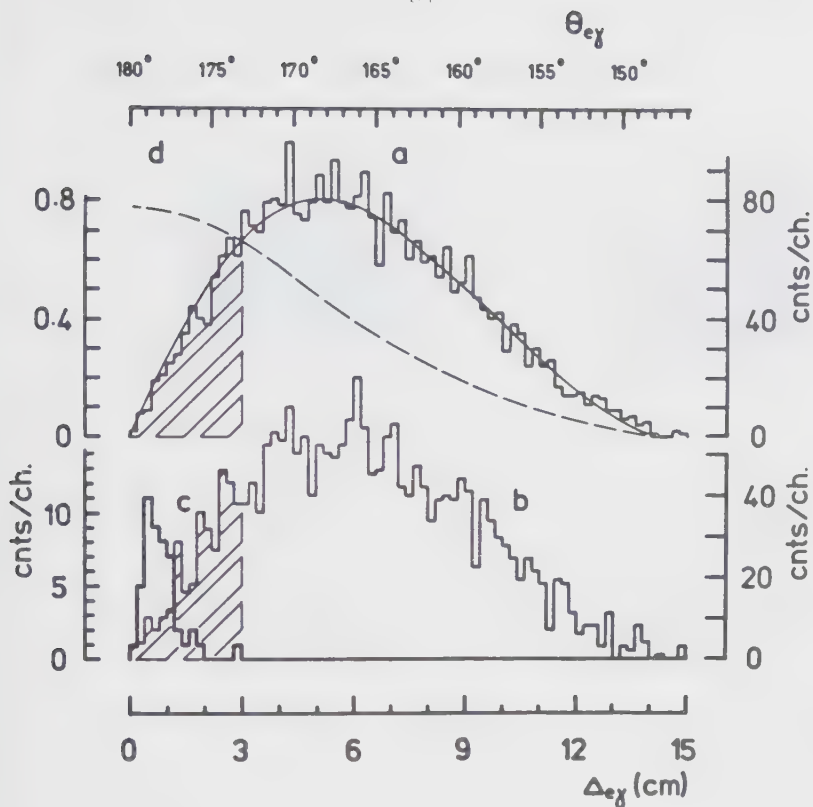


5. A : central part of the 2-dimensional distribution of the time differences Δt_{eT} (between S7 and T) and $\Delta t_{\gamma T}$ (between NaI $_{\gamma}$ and T) for energies $E_{NaI_e}, E_{NaI_{\gamma}} > 25$ MeV
- B, C: projections on Δt_{eT} and $\Delta t_{\gamma T}$ respectively
- D : projection on $\Delta t_{e\gamma} = \Delta t_{eT} - \Delta t_{\gamma T}$
- E : Δt_{eT} as a function of the target energy E_T
- F : $\Delta t_{e\gamma}$ as a function of $E_{NaI_{\gamma}}$

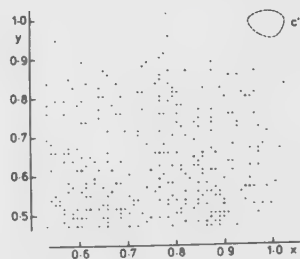
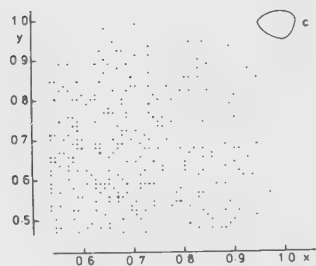
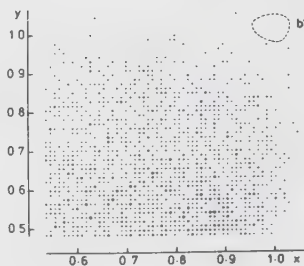
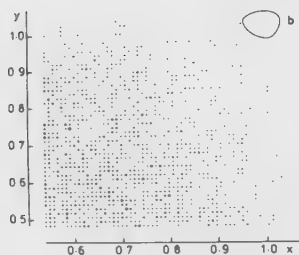
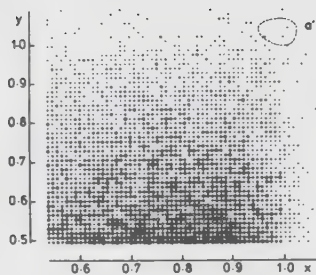
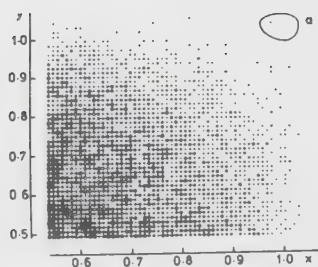
The solid lines in E and F define the prompt energy-dependent time windows. As an example the dashed lines in A show the defined prompt region for events with $E_T = 2$ MeV and $E_{NaI_{\gamma}} = 35$ MeV. All time scales are 1ns/div., the area of the dots is proportional to the intensity.

background originating from outside the target and the superior signature for positrons.

As a measure for the deviation from collinearity for converted events we define $\Delta_{e\gamma}$ as the distance from the measured hodoscope coordinates to the point of intersection of the extrapolated positron track with the hodoscope plane. $\Delta_{e\gamma}$ can be related in good approximation to the angle between positron and photon directions. Fig. 6 shows the $\Delta_{e\gamma}$ distributions for true and accidental coincidences for positron and photon energies above $\frac{1}{4} m_{\mu}$. Also indicated are the $\Delta_{e\gamma}$ calibration with cosmic muons and the angle dependent efficiency $\epsilon_{e\gamma}$ generated by a Monte Carlo simulation, taking into account the measured stop distribution in the target. One sees from this curve, that the probability to detect a collinear event, once the positron reaches S7, is 80%. Under the assumption of isotropic positron and photon distributions the $\Delta_{e\gamma}$ spectrum for accidental coincidences is described by $\epsilon_{e\gamma} \cdot \sin\theta_{e\gamma} \cdot \cos^2\theta_{e\gamma}$ (curve through fig. 6a) confirming the $\epsilon_{e\gamma}$ calculation. The $\Delta_{e\gamma}$ distribution for cosmic muons (fig. 6c) is determined by the resolutions in the wire chambers and the hodoscope only. The resolution for $\mu \rightarrow e\gamma$ events is broadened in addition by multiple scattering in the targets and by shower spread after the photon conversion. The efficiency of the applied collinearity cut of 30 mm, taking these effects into account, is estimated to be more than 99%.

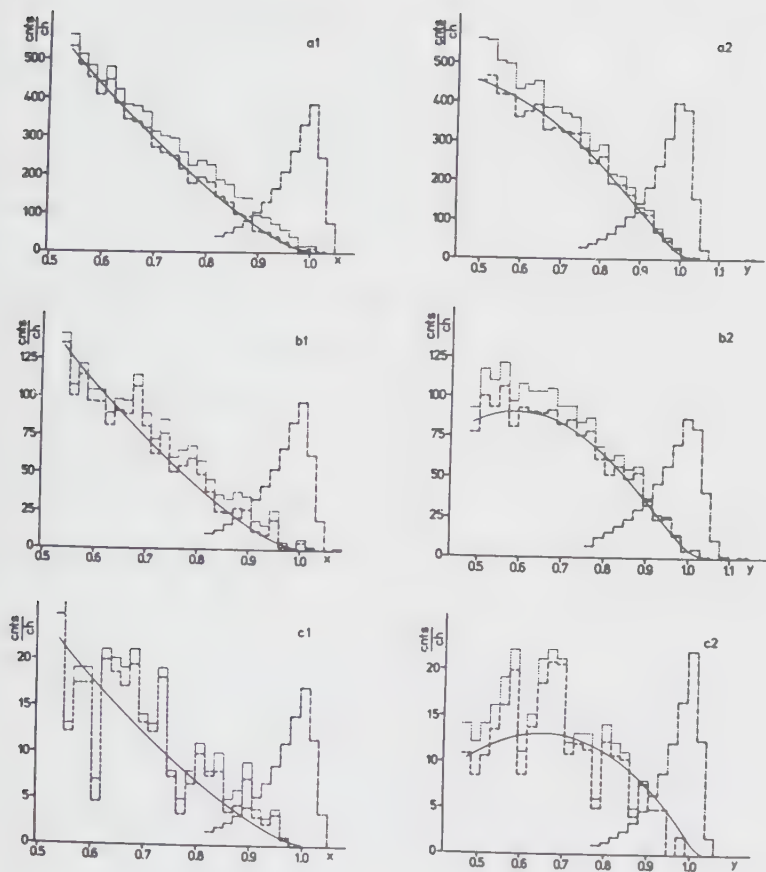


6. Distributions of accidental (curve a, right scale) and prompt (curve b, right scale) coincidences versus the collinearity parameter $\Delta_{e\gamma}$ defined in the text. Also shown is the distribution for cosmic muons (curve c, left scale) and the result of the Monte Carlo simulation for the acceptance for $e\gamma$ coincidences once the positron is detected (curve d, left scale). The curve through distribution a is derived from d by multiplication with $\sin\theta_{e\gamma}\cos^2\theta_{e\gamma}$. The shaded area has been estimated to contain more than 99% of the collinear events.



7. Two-dimensional energy distributions for prompt $e\gamma$ coincidences without γ -conversion (a), with complete γ -conversion (b) and those selected by the collinearity requirement (c) as defined in the text. a' , b' and c' are the corresponding distributions for accidental coincidences. Their contributions to the prompt spectra amount to 14%. Because of the collinear nature of the $\mu \rightarrow e\gamma$ decay that distinguishes it from the $\mu \rightarrow e\nu\bar{\nu}\gamma$ decay and accidental events, the process is detected in the more central parts of the crystals and produce therefore a larger energy signal. For this reason, the half maximum contours containing ~50% of the hypothetical $\mu \rightarrow e\gamma$ events are slightly shifted in a and b with respect to the point $x = y = 1$. For details see text. The area of the dots is proportional to the intensity.

The generated positron photon energy distributions and their projections are shown in fig. 7 and fig.8, respectively. Only energies above $\frac{1}{4} m_{\mu}$ are accepted, since the lower energy region is distorted by crosstalk from the positron detector to the photon detector. The positron and photon energies x and y are normalized to the maximum energy $m_{\mu}/2$. The spectra will be discussed in the next chapter.



8. Projections of the prompt energy distributions shown in fig. 7a,b,c on the positron energy x (a1,b1,c1) and the photon energy y (a2,b2,c2), dotted curves. The dashed curves are corrected for the accidental background shown in figure 7a', b', c'. The solid lines are fits of the theoretical $\mu^+ \rightarrow e^+ \bar{\nu}_e \bar{\nu}_\mu \gamma$ spectra using the response functions indicated by dash-dots at $x, y = 1$.

4. DATA ANALYSIS

The prompt spectra in fig. 7a, b, c are interpreted in a likelihood calculation considering three contributions: the radiative muon decay $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \gamma$, the hypothetical decay $\mu^+ \rightarrow e^+ \gamma$ and accidental coincidences. The accidental contribution is taken from the spectra 7a', b', c', taking into account the difference in time windows. The decay $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \gamma$ is calculated for pure V - A interaction (see Appendix) taking into account the total efficiency $\epsilon_{e\gamma}^{\text{tot}}(\theta, y)$ for the detection of a positron-photon pair and the response function of the detectors. In this way the measured rate for this decay can be used to determine the total number N_μ of muons stopped in the target. Independently N_μ is obtained from the rate of the normal decay $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$ and the total efficiency ϵ_e^{tot} for positron detection. Subsequently the sensitivity $S_{\mu \rightarrow e\gamma}$ of the measurement, i.e. the branching ratio corresponding to one $\mu^+ \rightarrow e^+ \gamma$ event in 7a, b, c, is found:

$$S_{\mu \rightarrow e\gamma}^i = \left[N_\mu \cdot \epsilon_{e\gamma}^{i, \text{tot}}(\theta=180^\circ, y=1) \right]^{-1}, \quad i=a, b, c \quad (1)$$

A detailed discussion of the efficiency is given in section 4.1, the determination of the response functions of the detectors and the energy calibrations are discussed in section 4.2 and the likelihood calculation is presented in section 4.3.

4.1. The efficiency

The overall efficiency for the detection of a positron-photon pair depends on its opening angle $\theta_{e\gamma}$ and the photon energy y and can be written:

$$\epsilon_{e\gamma}^{i,tot}(\theta_{e\gamma}, y) = \epsilon_e^{tot} \cdot \epsilon_{e\gamma}(\theta_{e\gamma}) \cdot \epsilon_i(y) \cdot \epsilon_{el} \cdot \epsilon_{tr} \cdot \epsilon_{\gamma} \cdot \epsilon_t, i=a,b,c$$

ϵ_e^{tot} is the total efficiency for positron detection, $\epsilon_{e\gamma}$ is the geometrical acceptance for a positron-photon pair once the positron is detected as introduced in chapter 3, $\epsilon_i (i=a,b,c)$ is the efficiency of the photon detector for photons of types a,b,c defined in chapter 3, ϵ_{el} is the efficiency of the logic electronics, ϵ_{tr} is the efficiency of the positron track selection, ϵ_{γ} is the efficiency of the photon signature selection and ϵ_t is the efficiency introduced by the chosen time windows.

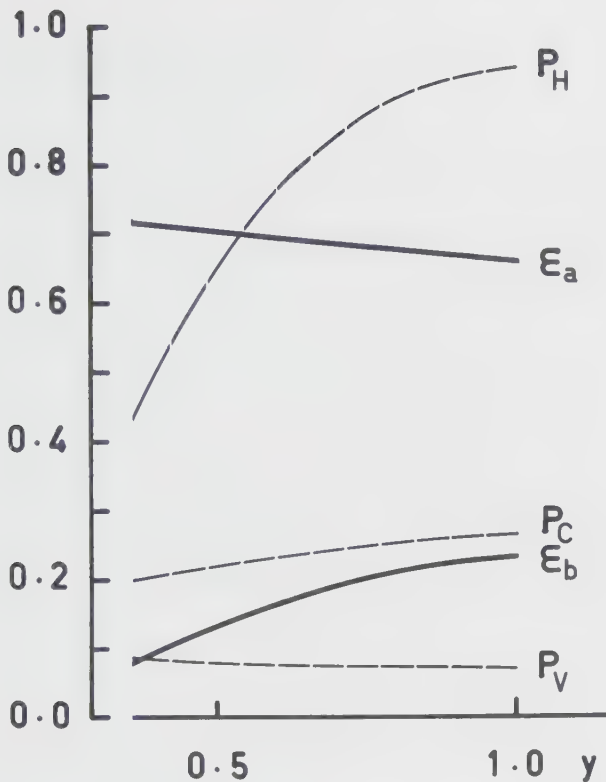
For the calculation of $S_{\mu \rightarrow e\gamma}$ from the rate of the radiative decay only $\epsilon_{e\gamma}$ and ϵ_i are considered since the other efficiency factors are the same for $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_{\mu} \gamma$ and $\mu^+ \rightarrow e^+ \gamma$. The processes $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_{\mu}$ and $\mu^+ \rightarrow e^+ \gamma$ have only ϵ_e^{tot} as common efficiency, thus an estimate of all other efficiencies is necessary to calculate $S_{\mu \rightarrow e\gamma}$ from the monitored rate of $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_{\mu}$.

The positron efficiency ϵ_e^{tot} is the product of the solid angle of the detector (1.9%), the intrinsic efficiency of the detector (mainly determined by the wire chamber efficiency of 95%;

the energy dependence is accounted for by the response function) and the efficiency of the logic electronics (90%, measured with the LED signals), resulting in $\epsilon_e^{\text{tot}} = 1.6\%$.

Fig. 9 shows the energy dependence of $\epsilon_a = (1-P_V)(1-P_C)$ and $\epsilon_b = (1-P_V)P_C P_H$. P_V is the probability for photon conversion in the target or the veto counters, P_C is the conversion probability in the converter and P_H is the probability for converted photons to cause a signal in the hodoscope. P_V and P_C are calculated and P_H is taken from the experiment. P_V , P_C and P_H are also shown in fig. 9. P_H is interpreted as the probability that the electromagnetic shower developed after the conversion in C is charged at the site of H. We could have increased ϵ_b by means of a thicker converter which, however, would have broadened the position resolution. For collinear events we take $\epsilon_c = 0.99 \epsilon_b$ (see chapter 3).

The efficiency ϵ_{e1} of the logic electronics for γ detection is normalized to the electronic efficiency for positron detection. ϵ_{e1} is equal to the ratio of the measured number of LED events to the number of LED monitor signals. Since the signals from the target, the positron detector and the photon detector are separately vetoed by the same veto counters, the dead time introduced by these vetos depends on the relative timing of the three detectors. As a consequence ϵ_{e1} depends on the position in the two-dimensional time distribution of fig. 5.



9. Detection efficiencies for photons without conversion (ϵ_a) and with complete conversion (ϵ_b) as defined in the text, as a function of the photon energy y . $\epsilon_a = (1-P_V)(1-P_C)$, $\epsilon_b = (1-P_V)P_C P_H$, where:

P_V : photon conversion probability in the target and the veto vounters S1-S4.

P_C : photon conversion probability in the converter C.

P_H : probability for photons that converted in C to give a signal in both planes of hodoscope H.

In particular ϵ_{el} is 85% for the prompt spectra 7a,b,c and 77% for the accidental spectra 7a', b', c'. The efficiency of the single track selection ϵ_{tr} is determined from the intensities of the spectra of fig. 4 and found to be 89%. The rejection of tracks that miss the targets (11% of all tracks) does not introduce a significant efficiency loss since a major part of them should be assigned to positrons that scattered into the positron solid angle from the collimator. We correct the monitored number of muon decays for this background. The photon efficiency ϵ_{γ} is introduced by the cuts of the definitions a) and b) of chapter 3. Rejected photon signatures are explained as mostly neutral pile-up. We find $\epsilon_{\gamma} = 90\%$.

The prompt time windows are chosen such that $\epsilon_t = 88\%$ ($= 0.9 \times 0.98$, see chapter 3).

The resulting total detection efficiencies for $\mu \rightarrow e\gamma$ are for unconverted events:

$$\epsilon_{e\gamma}^{a,tot}(\theta = 180^\circ, y = 1) = 0.51\% \text{ per } \mu^+ \text{ decay}$$

and for collinear converted events:

$$\epsilon_{e\gamma}^{c,tot}(\theta = 180^\circ, y = 1) = 0.18\% \text{ per } \mu^+ \text{ decay.}$$

4.2. The response functions

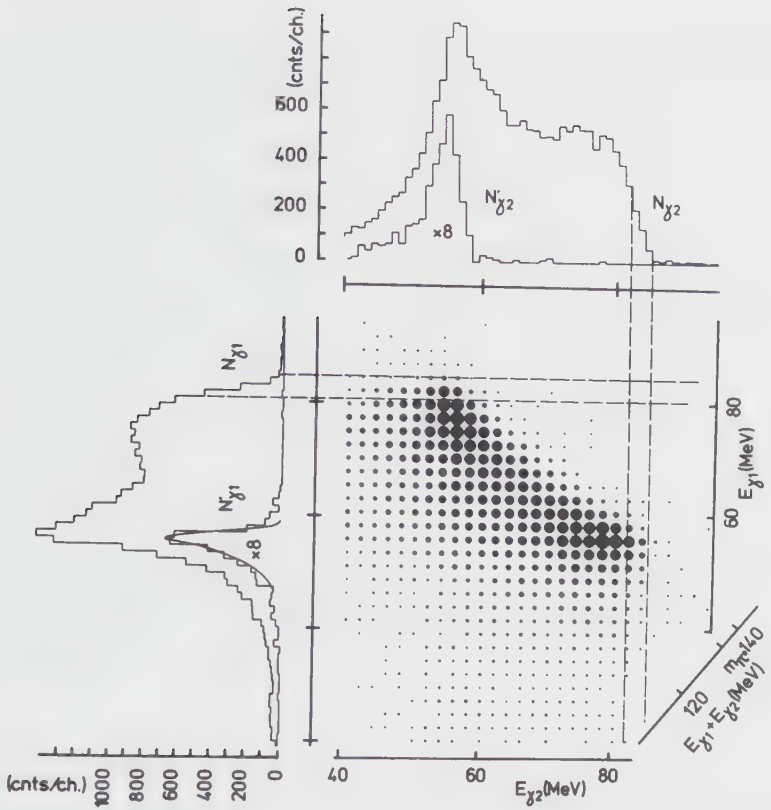
For the calculation of the $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \gamma$ spectrum and the $\mu^+ \rightarrow e^+ \gamma$ line shape a thorough understanding of the response functions of the positron and photon detectors is essential. A major contribution to the line shape in the energy region 25 - 53 MeV is related to shower losses out of the crystals. Since these losses depend on the impact position and angle of the detected particle, the response functions depend on the kinematics of the different processes: accidental coincidences, $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \gamma$, $\mu^+ \rightarrow e^+ \gamma$ and $\pi^0 \rightarrow 2\gamma$. The relevant kinematical parameter in the experiment is the deviation from collinearity $\Delta_{e\gamma}$. The first two processes have very similar $\Delta_{e\gamma}$ distributions (fig. 6a,b) because of the nearly isotropic angular distribution of the radiative decay in the analysed energy region. The decays $\pi^0 \rightarrow 2\gamma$ (for the applied energy cut) and $\mu^+ \rightarrow e^+ \gamma$ have collinear distributions and therefore $\Delta_{e\gamma}$ distributions that are very similar to that of the calibration with cosmic muons (fig. 6c). The effect on the response function is largest for the photon detector because of the distance between the crystal surface and the collimator ring defining the solid angle.

We measured the response functions directly with a well-focussed positron beam in the energy region 30 - 70 MeV. The relative resolution is only weakly energy-dependent in this region but varies from 5% FWHM in the center to 10% at the edge of the

accepted entrance area. The corresponding shift of the position of the maximum is -4%. The effective energy resolution of the positron detector during the measurement has been fitted to the $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$ spectrum and varies from 6% in the center to 10% at the edge. The slight increase with respect to the measured numbers is explained by the uncertainty in the energy loss in the target.

Fig. 8 shows the projections of the prompt energy distributions 7a,b,c on the positron energy (a1,b1,c1) and on the photon energy (a2,b2,c2) with and without correction for the accidental background. Also shown are fits of the $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \gamma$ theory using the indicated response functions. The positron response functions in a1 and b1 are fitted to the $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$ spectrum of the complete $\Delta_{e\gamma}$ distribution (FWHM: 4.5 MeV), the one in c1 to the $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$ spectrum of fig. 4E (FWHM: 4.3 MeV). This spectrum of events with tracks through the photon collimator simulates the spectrum for $\Delta_{e\gamma} \sim 0$ cm. The widths and positions of the photon response functions in a2 and b2 are treated as free parameters in the fits (FWHM a2 : 5.0 MeV, b2 : 6.0 MeV). The response function in c2 is taken from the $\pi^0 \rightarrow 2\gamma$ calibration for converted photons (FWHM: 3.9 MeV).

The result of the $\pi^0 \rightarrow 2\gamma$ measurement for photons that did not convert (type a) is shown in fig. 10. Monochromatic π^0 's of 2.9 MeV kinetic energy are produced by stopping π^- 's in a LiH



10. $\pi^0 \rightarrow 2\gamma$ calibration.

The central part shows the 2-dimensional distribution of the photon energies in NaI_{γ} ($E_{\gamma 1}$) and in NaI_{θ} ($E_{\gamma 2}$). $N_{\gamma 1}$ and $N_{\gamma 2}$ are the projections on $E_{\gamma 1}$ and $E_{\gamma 2}$ respectively. $N'_{\gamma 1}$ and $N'_{\gamma 2}$ are the projections of the events within the dashed windows. Their ordinates are enlarged by a factor of 8. The curve through $N'_{\gamma 1}$ is the fitted response function.

target through the reaction $\pi^- p \rightarrow \pi^0 n$. The decay photons have energies between 54.9 MeV and 83.0 MeV. The angle between the two photons varies from 156.4° for photons with equal energies to 180° for photons with extreme energies. By a selection of photons of maximum energy in one of the crystals, photons with energies just above 54.9 MeV are singled out in the other crystal in a collinear configuration like $\mu^+ \rightarrow e^+ \gamma$. The energy windows in fig. 10 demonstrate this procedure. The indicated response function has been fitted to a larger part of the spectrum (FWHM: 3.3 MeV).

For $\mu^+ \rightarrow e^+ \gamma$ we assume a positron response function as given in fig. 8c1 and photon response functions as found in the $\pi^0 \rightarrow 2\gamma$ calibration.

4.3. The likelihood calculation

We interpret N_{kl}^i ($i = a, b, c$), the measured energy distributions of fig. 7a, b, c, as the sum T_{kl}^i of three components:

$$T_{kl}^i = \alpha R_{kl}^i + \beta A_{kl}^i + \gamma M_{kl}^i, \quad i = a, b, c$$

The indices k and l refer to the channel numbers of x and y in 7a, b, c. R_{kl}^i is the spectrum of the radiative muon decay as calculated in the appendix folded with the efficiency functions $\epsilon_{e\gamma}(\theta_{e\gamma})$ and $\epsilon_i(y)$ and with the response functions given in

section 4.2. A_{kl}^i is the accidental background calculated from $N_{kl}^{i'}$, the measured energy distributions of $7a', b', c'$:

$$A_{kl}^i \propto \sum_l N_{kl}^{i'} \cdot \sum_k N_{kl}^{i'}, \quad i = a, b, c$$

We use A_{kl}^i instead of $N_{kl}^{i'}$ because of its higher statistical accuracy. M_{kl}^i are the hypothetical $\mu^+ \rightarrow e^+ \gamma$ line shapes given in section 4.2. The distributions are normalized such that:

$$\sum_{kl} R_{kl}^i = \sum_{kl} A_{kl}^i = \sum_{kl} M_{kl}^i = 1, \quad i = a, b, c$$

in the processed energy region, so α^i , β^i and γ^i are the expectation values for the number of events in $7a, b, c$. We assume the probability of the set $(\bar{\alpha}^i, \bar{\beta}^i, \bar{\gamma}^i)$ of the form:

$$\mathcal{L}^i = \mathcal{L}^i(\bar{\alpha}^i, \bar{\beta}^i, \bar{\gamma}^i) = \prod_{kl} [\exp\{-T_{kl}^i(\bar{\alpha}^i, \bar{\beta}^i, \bar{\gamma}^i)\} / N_{kl}^i] \\ \{ T_{kl}^i(\bar{\alpha}^i, \bar{\beta}^i, \bar{\gamma}^i) \}^{N_{kl}^i}, \quad i = a, b, c$$

In the first step α^i , β^i and γ^i have been treated as free parameters and their values are determined by searching for the maximum value of $\mathcal{L}^i$. The β^i values fitted this way are within their statistical accuracy consistent with the number of events in $7a', b', c'$ corrected for the differences in time windows and electronic deadtime of the prompt and accidental spectra. They are fixed at their calculated values in the final fits.

TABLE II

	a	b	c
α_{fit}	5841 ± 83	1460 ± 41	245 ± 17
β_{calc}	1079 ± 12	262 ± 5	47 ± 3
γ_{fit} (90% confidence)	<4.1		<2.5
δ	8.5×10^9	8.5×10^9	8.5×10^9
$R_{\mu \rightarrow e \nu \bar{\nu} \gamma}$	9.7×10^{-7}	2.4×10^{-7}	5.0×10^{-8}
$N_{\mu\delta}$	6.0×10^{11}	6.0×10^{11}	6.0×10^{11}
$N_{\mu\alpha}$	6.3×10^{11}	6.3×10^{11}	5.2×10^{11}
$S_{\mu \rightarrow e \gamma}$	3.1×10^{-10}	8.7×10^{-10}	8.8×10^{-10}
$R_{\mu \rightarrow e \gamma}$ (90% confidence)	$<1.25 \times 10^{-9}$		$<2.2 \times 10^{-9}$

Table II

Number of events and branching ratios of the distributions of
fig. 7a,b,c:

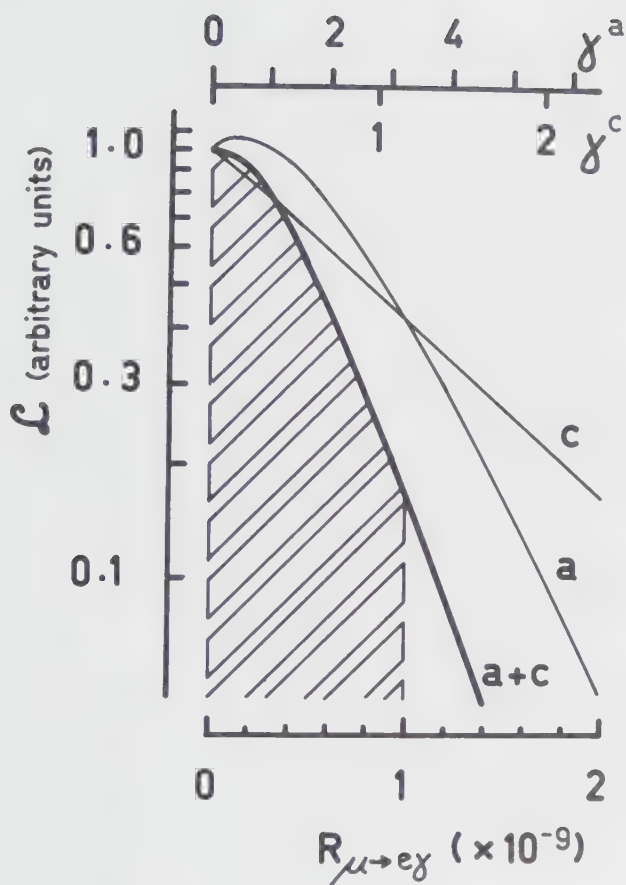
α_{fit}	: fitted number of $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \gamma$ decays.
β_{calc}	: number of accidental coincidences as calculated from the distributions 7a',b',c'.
γ_{fit}	: fitted number of $\mu^+ \rightarrow e^+ \gamma$ decays
δ	: monitored number of $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$ decays corrected for scattered positrons.
$R_{\mu \rightarrow e \nu \bar{\nu} \gamma}$	: calculated rate for $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \gamma$ in the processed energy region per muon decay in the positron solid angle.
$N_{\mu \delta}$	: number of stopped muons calculated from δ .
$N_{\mu \alpha}$	: number of stopped muons calculated from $R_{\mu \rightarrow e \nu \bar{\nu} \gamma}$ and α .
$S_{\mu \rightarrow e \gamma}$	: branching ratio for $\mu^+ \rightarrow e^+ \gamma$ if $\gamma = 1$.
$R_{\mu \rightarrow e \gamma}$	: branching ratio for the decay $\mu^+ \rightarrow e^+ \gamma$.

5. RESULT AND DISCUSSION

Table II lists the values of quantities that are relevant in the analysis. The event numbers α^i , β^i and γ^i have been introduced in the previous section. The monitor number δ of normal muon decays in the targets seen by the positron detector has been accumulated during the complete experiment. The rate of the radiative muon decay $R_{\mu \rightarrow e \nu \bar{\nu} \gamma}^i$ has been calculated for the geometry of the detection system (see Appendix) and was integrated over the processed energy region after folding with $\epsilon_i(y)$ and the response functions of the detectors. $R_{\mu \rightarrow e \nu \bar{\nu} \gamma}^i$ is normalized to the rate of the normal muon decay in the solid angle of the positron detector. The total number of muons that stopped in the targets has been deduced from δ and the efficiency $\epsilon_e^{\text{tot}} = 1.6\%$ for positron detection ($N_{\mu\delta}$) and from α^i , $R_{\mu \rightarrow e \nu \bar{\nu} \gamma}^i$ and the efficiency product $\epsilon_e^{\text{tot}} \cdot \epsilon_{e1} \cdot \epsilon_{tr} \cdot \epsilon_\gamma \cdot \epsilon_t = 1.0\%$ ($N_{\mu\alpha}^i$). For the calculation of the sensitivity of the measurement to $\mu^+ \rightarrow e^+ \gamma$ from expression (1) we took the mean of $N_{\mu\alpha}^i$ for $i = a, b, c$ since in this case only the efficiencies $\epsilon_{e\gamma}(\theta_{e\gamma})$ and $\epsilon_i(y)$ have to be taken into account (see section 4.1). The branching ratio for the decay $\mu^+ \rightarrow e^+ \gamma$ has been calculated from:

$$R_{\mu \rightarrow e \gamma}^i = \gamma^i \cdot S_{\mu \rightarrow e \gamma}^i$$

Fig. 11 shows the likelihood distributions $\mathcal{L}^a$ and $\mathcal{L}^c$ as a function of γ and of $R_{\mu \rightarrow e \gamma}$. Both curves show no evidence for



11. Likelihood functions of the $\mu \rightarrow e\gamma$ branching ratio $R_{\mu \rightarrow e\gamma}$ for events without γ -conversion (a), with the collinearity requirements (c) as defined in the text and for their combination (a + c). The correspondence between $R_{\mu \rightarrow e\gamma}$ and the numbers γ^a and γ^c is shown by the upper scales. The area of the shaded region is 90 % of $\int_0^1 \mathcal{L} dR$.

the detection of a $\mu^+ \rightarrow e^+ \gamma$ decay. The shape of $\mathcal{L}^a$ and the upper limit $\gamma^a < 4.1$ (90% confidence) are typical for a measurement with non-negligible background. The result can be improved only proportionally to the square root of the measuring time. The shape of $\mathcal{L}^c$ and the upper limit $\gamma^c < 2.5$ (90% confidence) are close to the ideal situation of an exponential likelihood function with an upper limit of 2.3 events typical for a measurement that is free of background. This result can still be improved linearly with the measuring time. Also shown in fig. 11 is the combined distribution $\mathcal{L}^{a+c} = \mathcal{L}^a \times \mathcal{L}^c$ as a function of $R_{\mu \rightarrow e \gamma}$. From

$$\int_0^{R(90)} \mathcal{L}^{a+c} dR_{\mu \rightarrow e \gamma} = 0.9 \int_0^1 \mathcal{L}^{a+c} dR_{\mu \rightarrow e \gamma}$$

with $R(90)$ the 90 % confidence limit for $R_{\mu \rightarrow e \gamma}$ we find

$$R(90) = 1.0 \times 10^{-9}.$$

Acknowledgement

The times that scientific research was done by single persons lie far back in history. This holds especially for experimental physics at particle accelerators, evolved over the past thirty years. The work described in this dissertation has been done in close collaboration with physicists from the University of Zurich, ETH and SIN.

First of all I would like to thank the others of the "hard core" of the collaboration: Hans Peter Povel, Claude Petitjean and Walter Dey. I deeply appreciate having had the opportunity to work for some years with three so distinct but (mostly) complementary individuals.

I am also very indebted to my advisor Roland Engfer, for the freedom I had on the one side and for his inspiring and very human approach to the experiment at the other. It is for more than one reason a pity that the oscilloscope pictures you made during the runs will never be published.

The experiment would not have been done the way it has been done without the skilfulness of Christian Walter. I especially enjoyed the very enlightening disputes we had nearly every week.

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APPENDIX A: The calculation of the $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \gamma$ spectrum

The differential branching ratio of the radiative μ -decay

$$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \gamma$$

has been calculated from a general formalism derived by Behrends, Finkelstein and Sirlin (39). Pure V-A interaction is assumed. We find

$$\frac{d^2R}{dx dy d\cos\theta} = \frac{\alpha}{8\pi} \frac{1}{y} \{ \Delta^{-1} F_{-1} + \Delta^0 F_0 + \Delta^1 F_1 + \Delta^2 F_2 - (1-\beta^2) (\Delta^{-2} h_{-2} + \Delta^{-1} h_{-1} + \Delta^0 h_0) \}$$

with the following definitions: $\Delta = 1-\beta\cos\theta$

$$F_{-1} = 8[y^2(3-2y)+6xy(1-y)+2x^2(3-4y)-4x^3]$$

$$F_0 = 8[-xy(3-y-y^2)-x^2(3-y-4y^2)+2x^3(1+2y)]$$

$$F_1 = 2[x^2y(6-5y-2y^2)-2x^3y(4+3y)]$$

$$F_2 = 2x^3y^2(2+y)$$

$$h_{-2} = 8[3(x+y)-2(x+y)^2]x$$

$$h_{-1} = 4[-3+4(x+y)]x^2y$$

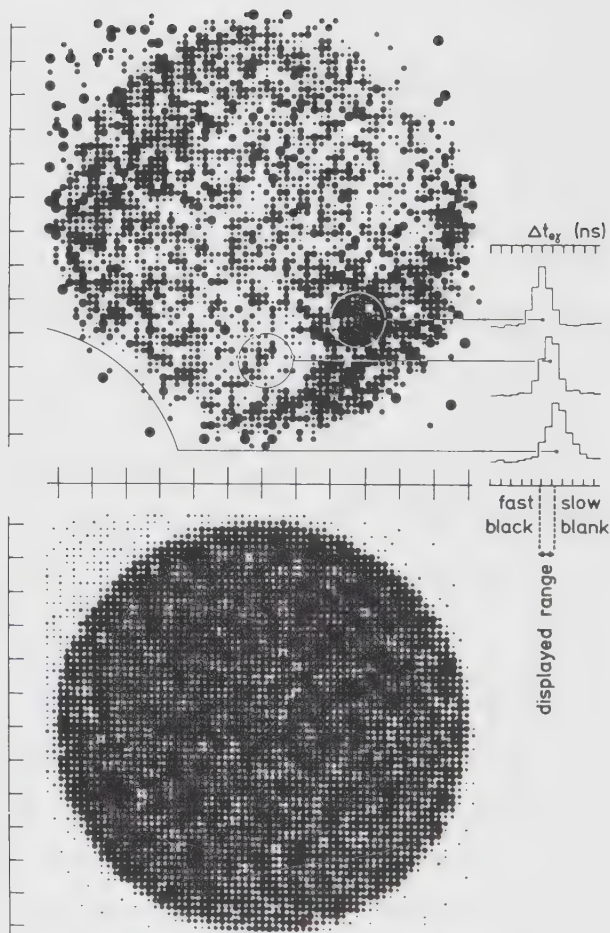
$$h_0 = -4x^3y^2$$

where x and y are the electron and the photon energies normalized to the maximum energy $m_\mu/2$, and θ is the angle between the electron and photon momentum.

The functions F_k agree with the definitions used by Fronsdal and Ueberall (40). The second term that is proportional to $(1-\beta^2)$ is omitted in ref. (40).

APPENDIX B: The geometry dependence of the counter S7 timing

The main background in the experiment is of accidental nature and therefore proportional to the width of the time window on $\Delta t_{e\gamma}$. For this reason it is important to improve the resolution of $\Delta t_{e\gamma}$ as much as possible. The photon timing is determined by NaI_γ and depends on the photon energy only. This correlation is accounted for by the energy dependent time window shown in fig.5. The positron timing is determined by S7 with leading edge discrimination. This means that the time signal is generated at the moment that the pulse from the photomultiplier of S7 exceeds a certain threshold. This moment strongly depends on the collection mechanism of the scintillation light that in its turn strongly depends on the positron impact position. The effect is demonstrated in fig.12. It shows the intensity distribution and the mean $\Delta t_{e\gamma}$ distribution over S7 for the decay $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \gamma$ with $E_e > 10$ MeV and $E_\gamma > 15$ MeV. To improve the statistical accuracy it would have been favourable to use Δt_{eT} for this study. However, Δt_{eT} is distorted by the fact that indirectly also the target timing depends on the impact position on S7, as will become clear in the discussion of the target energy loss (Appendix C). During the data processing Δt_{eT} has been shifted according to the distribution of fig.12 after a smoothing procedure to minimize the statistical fluctuations.



12. Lower part: intensity distribution of positrons from the decay $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \gamma$ over counter S7. Clearly visible in the left upper corner is an increase of the intensity caused by Cerenkov radiation in the plastic lightguide. Upper part: distribution of the mean value of the prompt $\Delta t_{e\gamma}$ timing for the same events. Also shown is the $\Delta t_{e\gamma}$ distribution for three regions of S7.

The symmetry of the timing pattern with respect to the direction of the light guide shows that it is of geometrical origin and not due to constructional imperfections. As should be expected the time signal in general is earlier towards the light guide, but this picture is completely distorted at the far end. This can be explained by the focussing effect of the circular scintillator that is probably also responsible for the sharp drops at the upper right and lower left edges. The widths of the three time spectra shown are mainly determined by the photon timing since the distributions are integrated over the energy region between 15 MeV and 53 MeV. The FWHM of the mean of the positron timing over S7 is 0.6 ns, that is, half of the FWHM of $\Delta t_{e\gamma}$ at $E_{\gamma}=53$ MeV. The correction is of particular importance since the 90% window defined on $\Delta t_{e\gamma}$ is quite sensitive to tails of the time distribution.

APPENDIX C: The positron energy loss in the targets.

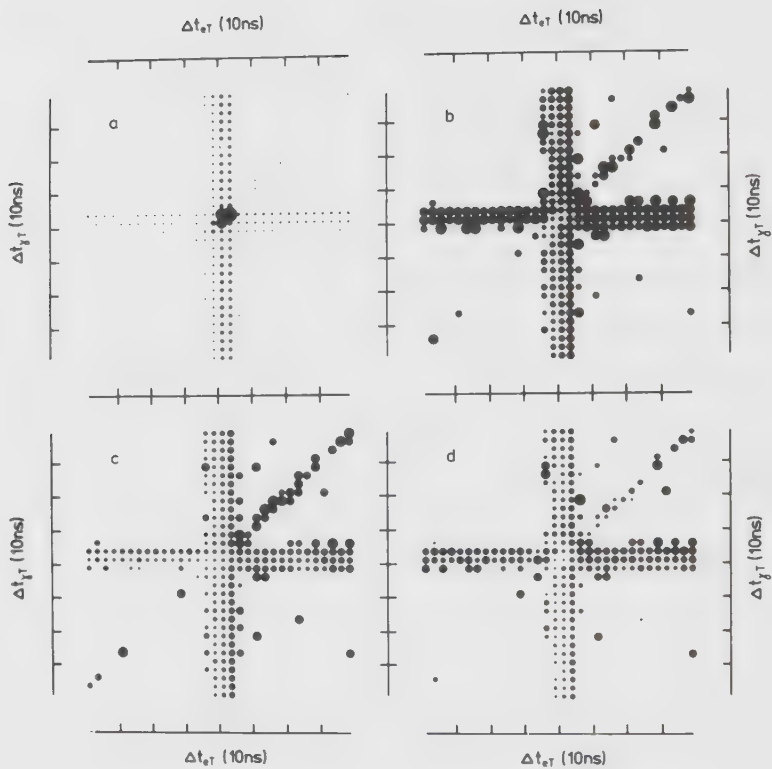
The energy loss of a relativistic positron in plastic scintillator material is about 2 MeV/cm. Since the positron path in the targets varies from 0-1.8 cm the uncertainty in the energy loss is about 3.6 MeV, which is a major contribution to the positron energy resolution. Unfortunately the energy resolution of the target counters is so poor that a direct use of E_T and $E_{T'}$ would even worsen the positron energy resolution. Furthermore E_T and $E_{T'}$ are contaminated by pile-up with a second decay positron in the case of accidental coincidences. The amount of pile-up depends on the relative timing of the two decays since the gates of both energy measurements are timed by T , thus ruining the assumption in the analysis that the accidental background is constant along Δt_{eT} and $\Delta t_{\gamma T}$ (apart from changes in the electronic dead time that affect the intensity but not the shape of the energy spectra along Δt_{eT} and $\Delta t_{\gamma T}$). The method used in the analysis partially avoids these problems. Two types of events are considered: with or without a signal in T' . The energy correction is calculated as a function of the angle α between the positron direction and the target orientation for each individual event:

$$E_{\text{targets}} = E_0 / \cos \alpha$$

E_0 is the mean energy loss for positron tracks that are perpendicular to the target surface and differs for both groups. It has

been fitted to the data. Also by this method accidental coincidences might be treated incorrectly but the amount of pile up turns out to be less than by the other method. To demonstrate this a closer look at the two-dimensional distribution of the positron, photon and target time signals is needed.

Fig.13 and fig.14 present four distributions over $\Delta t_{eT} \cdot \Delta t_{\gamma T}$ and its three projections on Δt_{eT} , $\Delta t_{\gamma T}$ and $\Delta t_{e\gamma}$. The parts a show the time spectra themselves, parts b show the probability distributions of the event signature with signals in both targets (as a measure of the target energy correction) and parts c and d show the mean values of E_T and E_{γ} . From 13a one learns that events with three uncorrelated time signals, distributed randomly in the $\Delta t_{eT} \cdot \Delta t_{\gamma T}$ plane, are at a negligible level. The large intensity in the middle is caused by events with three coincident time signals (real positron-photon coincidences originating in the targets). Events with only two correlated time signals have distributions along three straight lines. The vertical line consists of events of which the positron and the target signals are time correlated (Δt_{eT} prompt), the horizontal line consists of events for which the photon and the target signals are time correlated ($\Delta t_{\gamma T}$ prompt) and the diagonal consists of events for which the positron and the photon signals are time correlated ($\Delta t_{e\gamma}$ prompt). In the case that both the positron and the photon



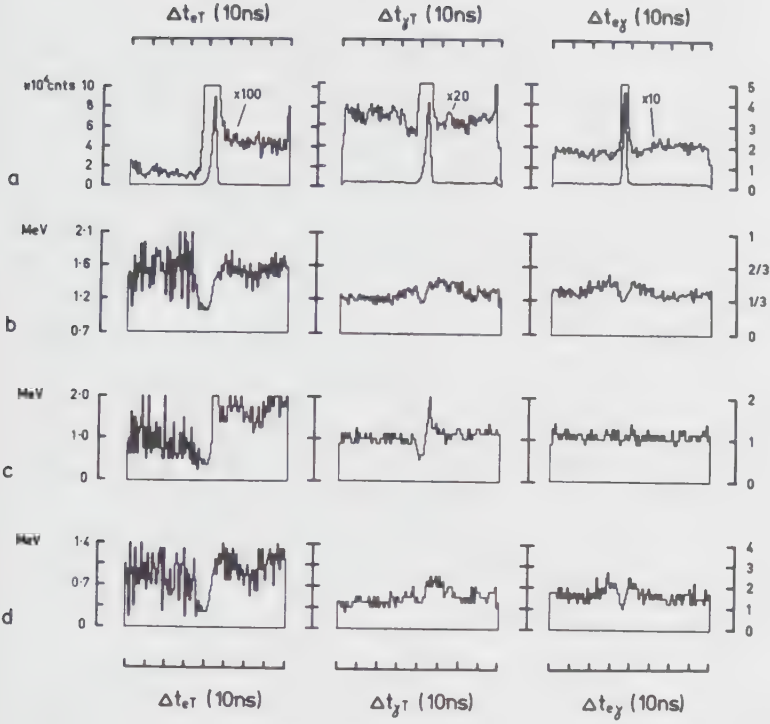
13. Distributions over $\Delta t_{eT} \cdot \Delta t_{\gamma T}$ for $E_e > 10$ MeV, $E_\gamma > 10$ MeV:

a: the number of events,

b: the probability that both targets have signals,

c: the mean value of E_T ,

d: the mean value of E_T ,.



14. Distributions over Δt_{eT} , $\Delta t_{\gamma T}$ and $\Delta t_{e\gamma}$ for $E_e > 10$ MeV,

$E_\gamma > 37$ MeV:

a: the number of events,

b: the probability that both targets have signals,

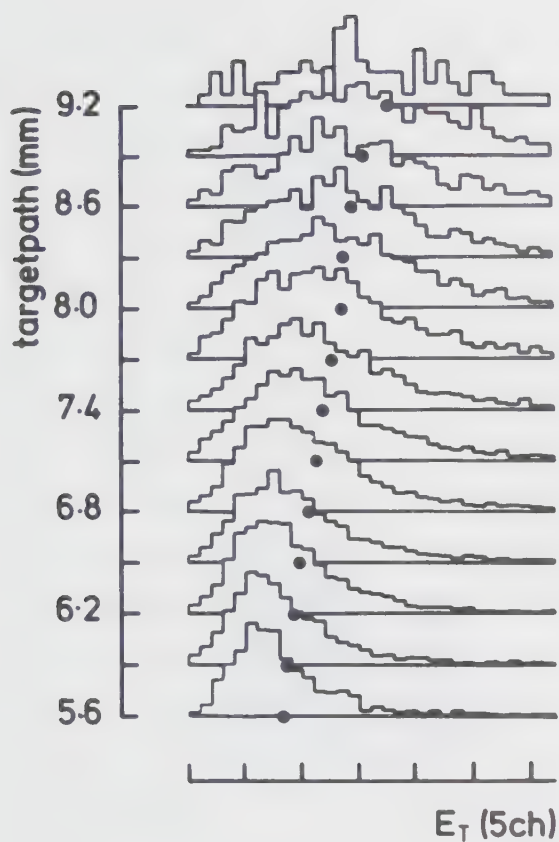
c: the mean value of E_T ,

d: the mean value of E_{T1} .

are time correlated with the target the circuitry takes the first target signal, thus those events can be situated on the "after prompt" half axes only. The large before-after asymmetry in Δt_{eT} as demonstrated in fig.14a can be explained under the assumption that about 75% of the positron signals are time correlated with the target. The same absolute intensity step of about 300/ch occurs in $\Delta t_{\gamma T}$, but the background of photons that are not target correlated is about 1300/ch, so only about 20% of the photons with an energy above 37 MeV are time correlated with counter T. This doesnot mean that only 20% of the high energy photons originates in the targets since only a small part of the photons from T' cause signals in T due to the fact that the positron accompanying the photon tends to go in the same direction as the photon.

The increased intensity in $\Delta t_{\gamma T}$ in the region where the photon signal comes at least 10 ns earlier than the target signal is probably caused by dead time of one of the veto signals.

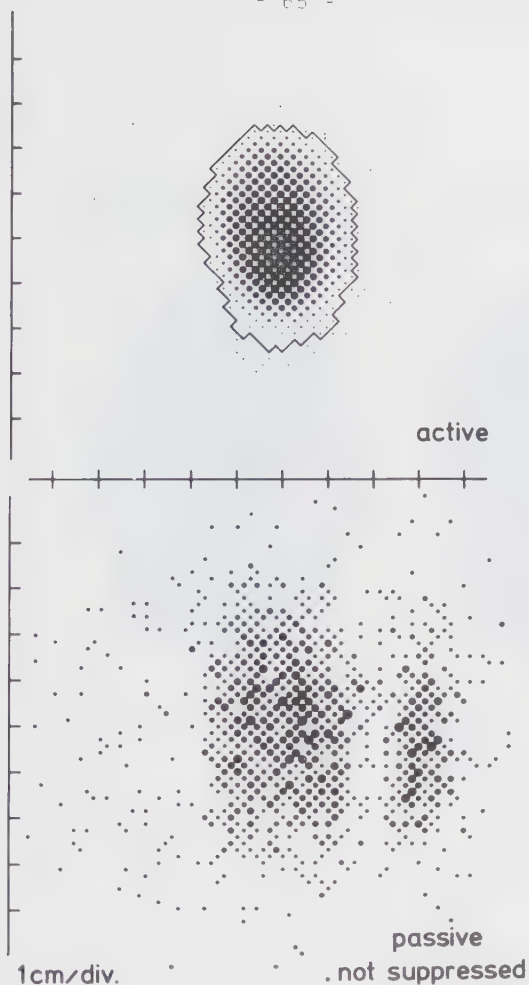
Quantitatively the target energy loss can be studied best from the spectra of fig.14. The normalization of c and d has been done on the prompt events in $\Delta t_{e\gamma}$ since these are free from pile up. The largest target signals are found in the after-prompt region of Δt_{eT} since here the pile up probability is 75%. The correction according to b however is less than half of the sum of c and d. The variation in the correction is



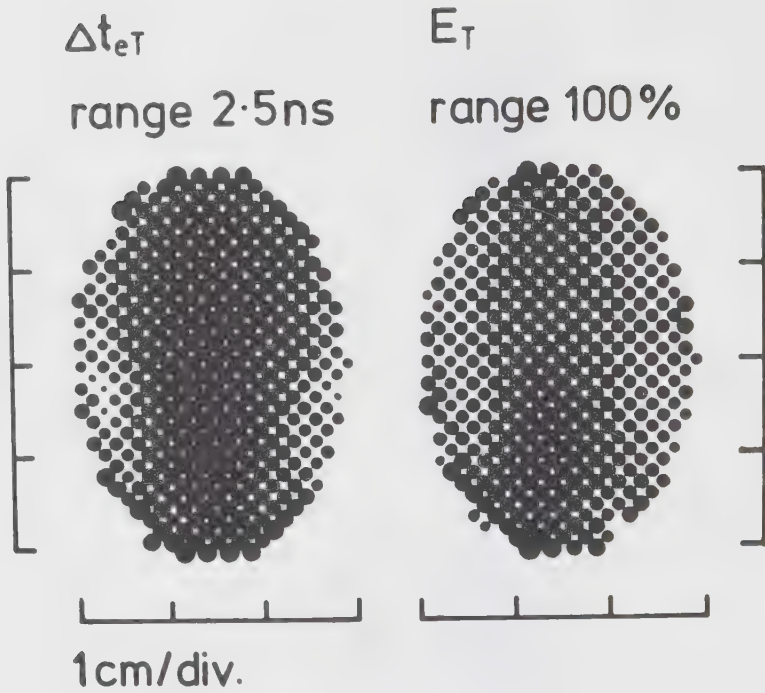
15. The E_T spectrum of decay positrons from T' for different values of the target path as computed from the angle between the target and the positron direction. The number of events has been renormalized to give the same maximum value for all spectra. The dots indicate the position of the mean value of E_T .

even five times smaller. In the distribution along Δt_{YT} the difference between the methods is smaller because of the larger target uncorrelated background. Here the variation in b is about half of that in the sum of c and d . The structure in Δt_{YT} of c is caused by the walk of the time discriminator of T . The shape of the pile up distributions in b and d is the superposition of the shapes of the energy gate signal (20 ns width, timed by T) and the photomultiplier signal of T .

The ionization loss in T of positrons from T' depends on the direction of the positron. The dependence of E_T on the length of the positron path in T is shown in fig.15. Though there exist a perfectly linear dependence between the length of the path and the mean value of E_T the distributions of E_T show a very poor resolution. This can be explained partially by the dependence of E_T on the position at T . The muon stop distribution in the active target and the passive material around it is shown in fig.16. The ring shaped distribution around the target comes from positrons that scattered from the positron collimator. In the distribution of passive stopping material one recognizes the target box and the end of the beam moderator. The flat background probably comes from muons that stopped at the surface of counter $S1$. The reduction of the "passive positron background" by the elliptically shaped contour is about 30%. Fig.17 shows the mean of E_T over the target. It varies within a factor of two. Also shown is the distribution of the target timing.

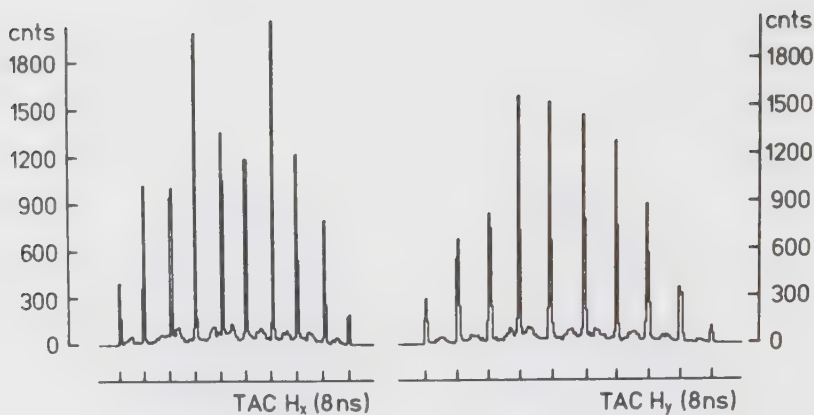


16. Muon stop distributions determined from the tracks of the decay positrons for different time regions.
- Upper part: Δt_{eT} prompt. The contour indicates the region accepted in the analysis.
- Lower part: Δt_{eT} before prompt. Notice the rectangular shape of the target box and the end of the moderator M.



17. Distributions over the target of the mean values of the prompt Δt_{eT} timing and E_T .

The position dependence of the time signal from T has not been corrected for as in the case of counter S7, since its amplitude dependence can be taken into account. This E_T dependence of the window on Δt_{eT} is shown in fig.5E.

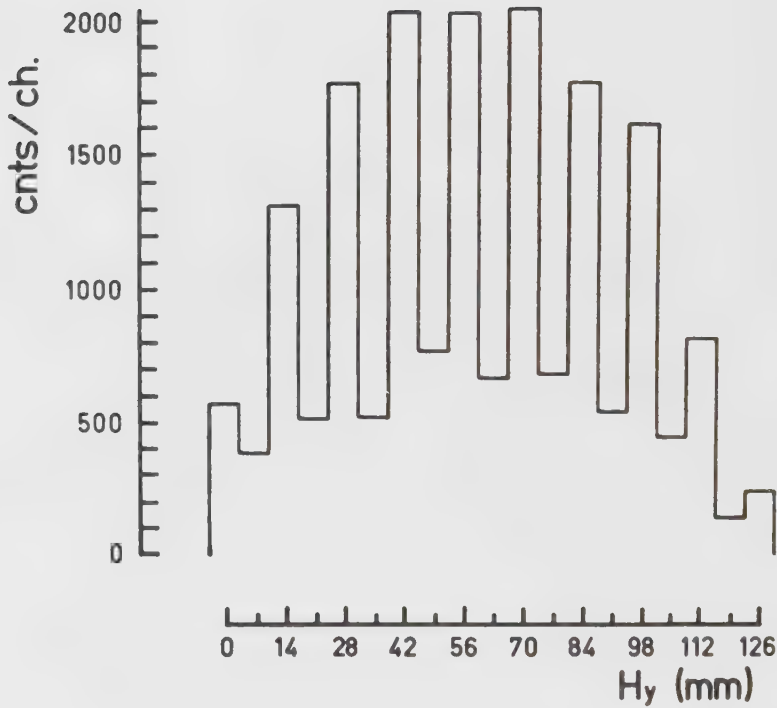


18. Hodoscope delay line spectra for $E_\gamma > 15$ MeV. Clearly visible is the difference in time resolution between events with one responding strip (start and stop with the same signal) and events with more than one responding strip (start and stop with different signals; their difference in amplitude causes a broadening of the time distribution because of walk in the timing discriminators).

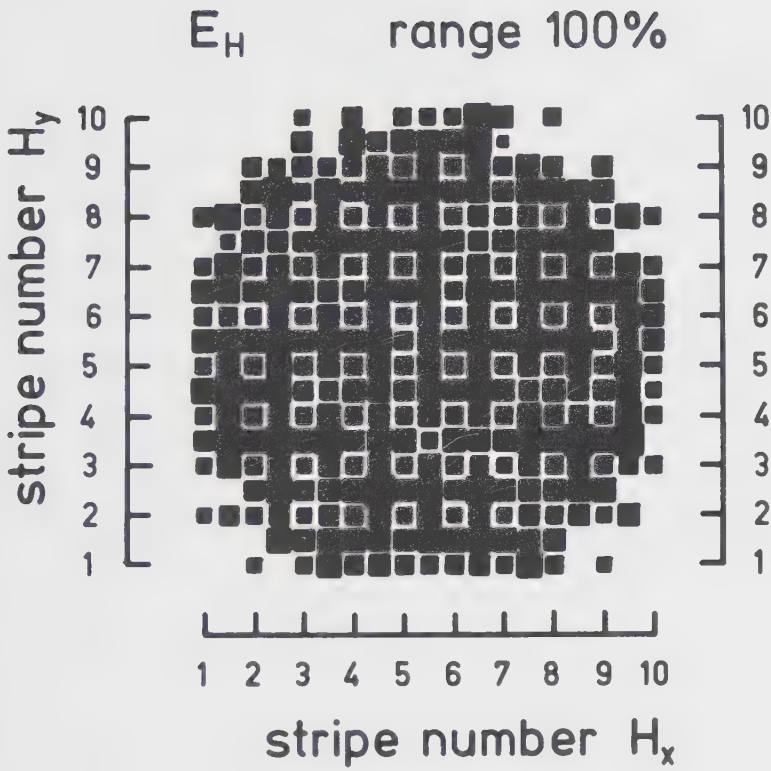
APPENDIX D: The hodoscope.

The hodoscope consists of two planes of ten plastic scintillator strips each ($140 \times 14 \times 3$ mm³ each). The energy signal E_H is the sum of the signals from all twenty strips added passively. The coordinate information has been converted into two time differences in the following way. All strips give two analog signals: a start and a stop signal. The start signals are delayed in steps of 4 ns increasing from the first strip to the tenth strip, the stop signals are delayed in 4 ns steps decreasing from the first strip to the tenth strip. The ten start signals are added and the ten stop signals are added. The time difference between these two signals is measured with a TAC. The two TAC spectra are shown in fig.18. Events with only one responding strip per plane show up as ten lines at 8 ns distances. If two or more strips have signals the circuit gives a time signal halfway the positions of the two outer strips. In the data processing the time signals are converted into position signals. The position spectrum of the y plane is shown in fig.19. Only nineteen positions occur: those of the ten strips and the nine intermediate positions.

Fig.20 shows the mean value of E_H over the hodoscope plane. The homogeneity is remarkable, especially compared to that of the target counter (see fig.17). Three types of hodoscope signatures can be distinguished, with different values for E_H :



19. The delay line spectrum of the y plane after conversion into a position spectrum. Since the photon primarily produces an e^+e^- pair there is a considerable probability of triggering two neighbouring strips and giving rise to intermediate positions.



20. Distribution of the mean value of E_H over the hodoscope plane. The corresponding intensity distribution is shown in fig.24.

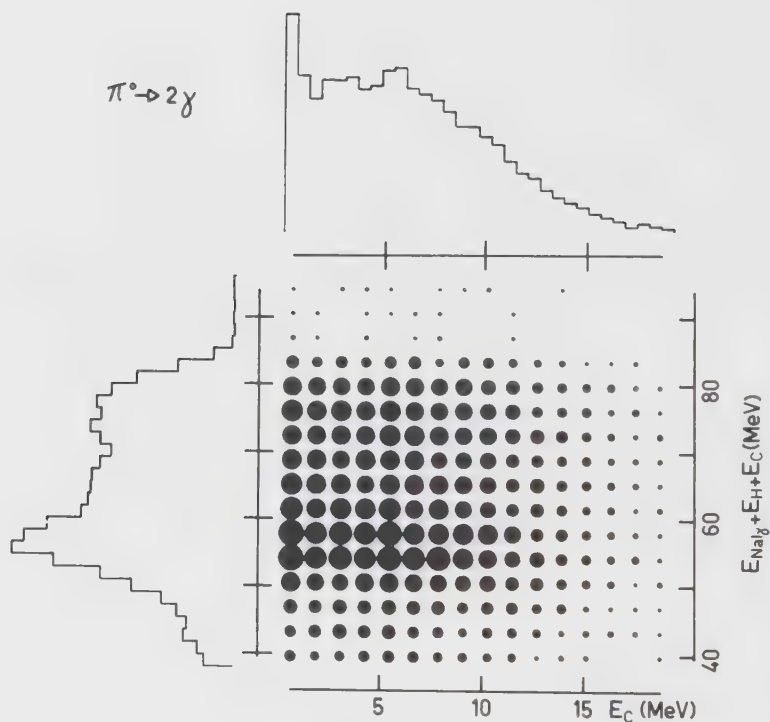
- one responding strip in each plane: $E_H \approx 1.8$ MeV
- more than one strip in only one plane: $E_H \approx 3.0$ MeV
- more than one strip in both planes: $E_H \approx 3.6$ MeV

The energies should be compared to the about 1.2 MeV ionization loss of a relativistic electron moving perpendicular to the hodoscope plane. The calibration of E_H has been done on the endpoint of the photon spectrum. The procedure is the same for the E_C calibration that will be discussed in Appendix E.

APPENDIX E: The converter.

The energy calibration procedure is demonstrated for the $\pi^0 \rightarrow 2\gamma$ measurement in fig.21. The calibration is chosen such that the half maximum positions at the two edges of the E_γ distribution do not depend on E_C . In a similar way E_C has been calibrated using the bremsstrahlung spectrum. In this case the calibration is chosen such that the E_γ endpoint is independent of E_C . The two methods are consistent within 2%.

The "endpoint" of the E_C spectrum is at about 20 MeV, which can be explained by the fact that the Landau distribution peaks at about 9 MeV for relativistic electrons going through C.

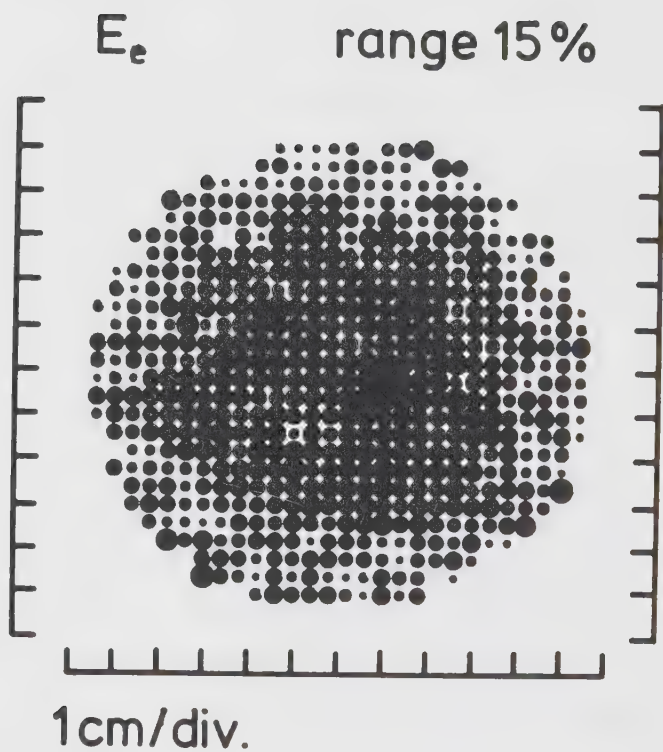


21. Intensity distribution over $E_C \cdot E_\gamma$ ($E_\gamma = E_{\text{NaI}_\gamma} + E_H + E_C$) and its projections for the $\pi^0 \rightarrow 2\gamma$ calibration. Notice the increase in the first channel of the E_C spectrum caused by 511 keV escape photons from NaI_γ .

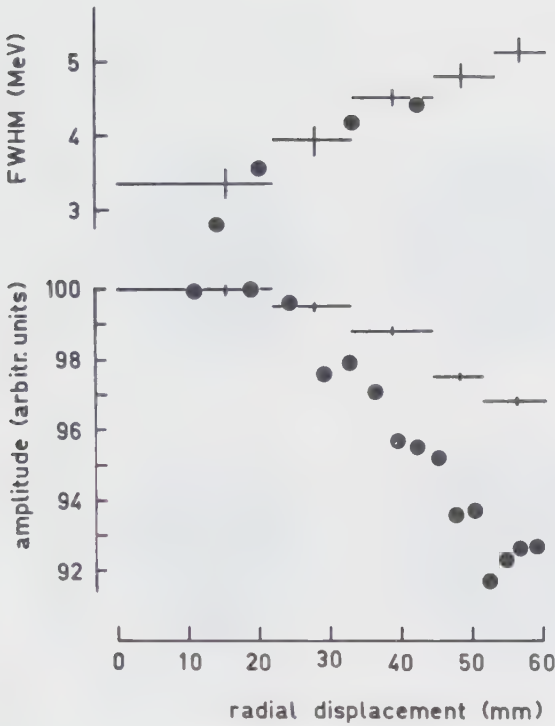
APPENDIX F: The geometry dependence of the NaI(Tl) response function.

The mechanism that causes a dependence of the NaI(Tl) response function on the impact position of the detected particle has been discussed already in some detail in chapter 6.2. Here a more quantitative treatment is presented. Three parameters of the response function will be considered: the FWHM, the position of the maximum and the mean value. The last two differ due to the asymmetric shape of the response function.

Measurements with a mono-energetic positron beam show that the shape of the response function is in good approximation independent of the energy in the region between 20 MeV and 53 MeV. In the fits to the Michel spectrum we assume a constant shape over the complete spectrum. Under this assumption the mean value of the 53 MeV response function has the same geometry dependence as the mean energy of the $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$ spectrum (~ 32 MeV): The latter is shown in fig.22. As should be expected the distribution is radially symmetric. For this reason the response function will be studied in the following as a function of the radial displacement at the site of counter S7. The results of the fits and the direct measurements are shown in fig.23. Both the width and the amplitude appear to depend roughly linearly on the radial displacement. For this reason their mean values over S7 are equal to the values at the mean



22. Distribution of the mean value of E_e for the decay $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$ over the wire chamber WC2.



23. Radial distribution over S7 of the width and the amplitude of the NaI_e response function for $E_e = 53$ MeV.

Upper part: FWHM fitted to the $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$ spectrum

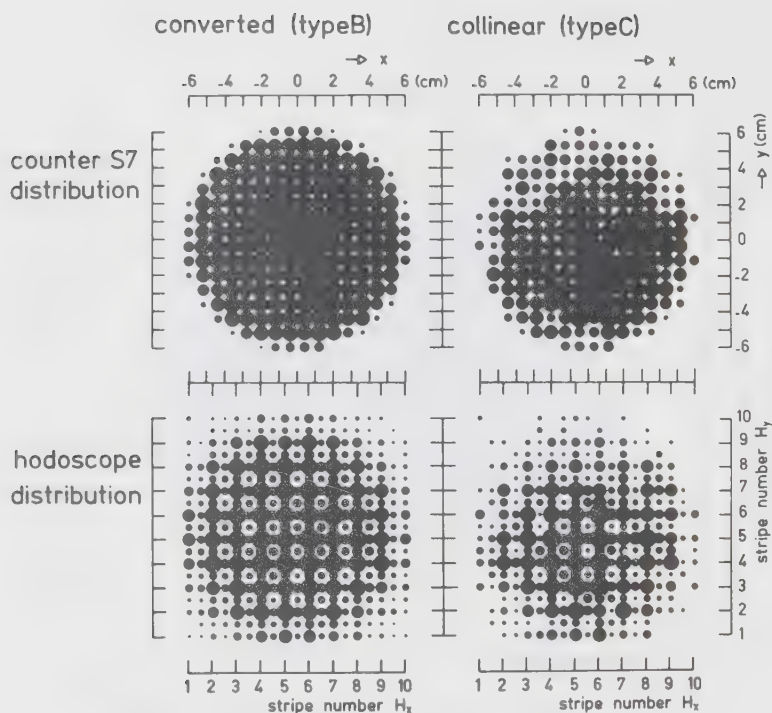
(crosses) and as directly measured with a mono-energetic positron beam (dots).

Lower part: position of the maximum of the response function

fitted to the $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$ spectrum (crosses) and the mean

value of E_e for the decay $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$ (dots). The difference

in slope is due to the asymmetric line shape.



24. Intensity distributions over counter S7 and hodoscope H for events of type b and c. Notice the concentrated irradiation for the collinear events that causes a different response function with increased amplitude and decreased FWHM compared to b.

radial displacement. This mean displacement on its turn depends on the kinematics of the detected process. It is equal to 40 mm for uncorrelated or isotropically distributed positron-photon coincidences, it is about 37 mm for collinear particles. The influence of the kinematics on the irradiation patterns of both detectors is demonstrated in fig.24. The slight deviation of the intensity maximum for collinear events from the center of the detectors can be explained by the asymmetric stop distribution (see fig.16).

APPENDIX G: Experimental limits on the branching ratios of
some muon number violating processes.

Table III gives the results of recent searches for the processes
 $\mu^+ \rightarrow e^+ \gamma$, $\mu^+ \rightarrow 3e$, $\mu^- A \rightarrow e^- A^*$, $\mu^- A \rightarrow e^+ A^*$ and $\mu^+ \rightarrow e^+ \gamma \gamma$.

process	laboratory	year	br.ratio	reference
$\mu^+ \rightarrow e^+ \gamma$	TRIUMF	1977	$<3.6 \times 10^{-9}$	(7)
	SIN	1977	$<1.0 \times 10^{-9}$	(8), (10)
	LAMPF	1979	$<1.8 \times 10^{-10}$	(9)
$\mu^+ \rightarrow 3e$	Dubna	1976	$<1.9 \times 10^{-9}$	(41)
$\mu^- A \rightarrow e^- A^*$	SIN	1977	$<7 \times 10^{-11}$	(42)
$\mu^- A \rightarrow e^+ A^*$	SIN	1979	$<2 \times 10^{-10}$	(43)
$\mu^+ \rightarrow e^+ \gamma \gamma$	L. Berkeley	1974	$<4 \times 10^{-6}$	(44)
	TRIUMF	1978	$<5 \times 10^{-8}$	(45)

Table III: Recent experimental results on muon number violating
processes

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CURRICULUM VITAE

Name	Andries van der Schaaf
Heimat und Staatsangehörigkeit	die Niederlande
Geburtsdatum	26. März 1952
Geburtsort	Amsterdam
1964 - 1969	Ir. Lely Lyceum, Amsterdam
1969	Maturität, Typus hbs B
1969 - 1975	Instituut voor Kernfysisch Onderzoek (IKO), Amsterdam
1975	Diplom in Experimentalphysik
1975 - 1980	Assistent am Physik-Institut der Universität Zürich

Dozenten bei denen ich Vorlesungen besuchte in Amsterdam:

J. de Boer, A. Botzen, T.J. Dekker, D. van Hulst, S.R. de Groot,
J. Hemelrijk, J. Hilgevoord, H.F. Jongen, P.F.A. Klinkenberg,
J.C. Kluyver, H.A. Lauwerier, C.G. Lekkerkerker, R. van Lieshout,
F.A. Muller, P.S. The, N.J. Trappeniers, C. de Vries, G. de Vries,
A.H. Wapstra und S.A. Wouthuysen.

Dozenten bei denen ich Vorlesungen besuchte in Zürich:

E. Brun, R. Engfer, M.P. Locher, V. Meyer und N. Straumann.

Die Dissertation ist unter der Leitung von Prof. Dr. R. Engfer
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